Math 5378, Summer I 2009: Concepts of Geometry in K–8 Mathematics

selected days and times, Carter Junior High, AISD

Instructor: Dr. Christopher Kribs Zaleta
Office: 483 Pickard Hall
Phone: (817)272-5513, fax 272-5802
email: kribs@uta.edu
WWW: http://mathed.uta.edu
Office Hours: Wednesdays 11 AM–12 noon and by appointment

Prerequisites: graduate standing and consent of instructor
Text materials: DMI's *Examining Features of Shape* casebook (henceforth EFS).
Additional materials will be provided in class or on the course web page.
Course home page: http://mathed.uta.edu/kribs/5392e.html
Last day for withdrawal: June 21

Class policy on drops, withdrawals, academic honesty, and accommodating disabilities follows the University policy on these matters. Copies can be obtained upon request.

LEARNING OUTCOMES: After completing this course, students should be able to:

- identify and use appropriate representations of geometric ideas in teaching situations, including conceptual, contextual, concrete, pictorial, and symbolic models
- analyze student thinking involving geometric ideas
- design and implement research-based lessons to teach geometry concepts, including the development, selection and sequencing of problems
- identify and use different geometric representations, as well as examples and counterexamples, to make definitions of terms in geometry
- recognize and apply geometric notions of comparison, including congruence, similarity, symmetry, scaling, and transformations such as translation, rotation, and reflection
- recognize and apply different meanings of angle
- construct and analyze different two-dimensional representations (such as nets, cross sections, and projections) of three-dimensional objects, and distinguish their properties

FORMAT: This course will study geometry concepts in several ways: through work on challenging mathematical problems to develop our own geometrical abilities (and communicating the results, to develop our expository abilities); discussing what research has discovered about the learning of geometry concepts at the K–8 levels; and examining specific instances of K–8 students' geometrical work, through both case studies and our own classroom practice.

Before class each week you will read articles and/or case studies from K–8 mathematics, and make notes on them in preparation for class discussions. You will also often work on mathematics problems outside of class, to facilitate their discussion in class. We will typically begin class by working on new mathematics problems and discussing their solutions, in both small and large groups. We will follow this up by discussing the assigned readings, as well as other related topics. We will typically end class with time for reflection on how the topics we have discussed apply to our own classrooms.

During class discussions we will often refer back to work we have done earlier in the course, so please bring your notes and papers from previous sessions to class.

POLICIES:

• Students who are not classroom teachers will need to make arrangements to interact with K–8 students for many of the assignments (those starred * on the calendar).

• Students are expected to be on time, prepared and ready to work every week. This class meets 5:00–8:00 PM on 5/26, 5/28, 6/02 and 6/08, and 3:30–6:30 PM Mon-Thu for the rest of June. Each student is allowed the equivalent of one week's absence (3 total hours) for whatever reason without penalty. All subsequent absences (including arriving significantly late) will result in the reduction of the final course grade by one-half letter grade (5%) for each absence. See DMI handout for policy on making up work from a missed class.

• With the exception of examples of student work, written assignments are expected to be typed and use correct grammar and punctuation. (Diagrams, equations, etc. may of course be hand-drawn.)

• Each student is allowed one late submission during the semester. The paper must be submitted before the beginning of the class period following that in which it was due. Papers not submitted by the end of class time on the due date are considered late. Submission of a late paper constitutes the student's agreement that this is the one allowed late assignment.

• Each student is allowed one electronic submission during the semester. Electronic submissions must be complete and not missing any ancillary materials such as student work necessary for grading. (If the electronic submission is made late, then it is both the only late paper allowed and the only electronic submission allowed.) This does not include drafts sent for consultation prior to submission, but consultation *must* take place in person or via telephone.

• Each student is allowed to submit one revised paper for a regrade, under the following terms: The revised paper and the graded original must be turned in together at the penultimate class meeting. The new grade replaces the original. Students are encouraged to consult with the instructor prior to submitting a revised paper.

GRADES: Your grade for the course will be determined by five elements, each of which has equal weight: (1) journal entries and participation, (2) a written student interview, (3) a short case study, (4) a paper detailing your own mathematical work, and (5) a lesson involving a problem you select to develop geometrical concepts in your students. All of these are detailed in the next section.

Note on study time: Summer courses take a sixteen-week semester and compress it into five weeks. That's a compression factor of more than three! Not only does this time compression leave students less time to "unpack" and reflect between class meetings, but it also means that in order to engage fully in the course, one has to spend more than five times as many hours per day outside of class on coursework than would be true during a long semester. In particular, instead of the usual rule of thumb of six hours per week outside of class for every three hours per week spent in class, the proportion becomes 18 hours per week out of class for 9 hours per week in class. This is equivalent to a half-time job. Please be careful to plan accordingly.

Assignments

1. Journal

On days in which there is not a major assignment due, you will write a short (about one page) reflection in response to a prompt given below. Many of the prompts are also given in the DMI handouts distributed in class. Some involve "action research" reports in which you will write about your own students' mathematical work. You will often use and discuss your responses in class, within your small groups and in large group. These reflections are to be turned in at the end of class; I will respond to them in writing and return them at our next class meeting. Grading will be limited to verifying that responses are appropriate in topic and scope (length).

Your journal entries will serve to document your preparation for class each day (and your growth over time); your preparation and participation grade will be based half on your journal (entries should be complete each day before class) and half on your participation in class discussions (I expect participation in large-group discussion at least ten of the fifteen times we will meet).

J1 *Key ideas.* What are the key ideas of geometry that you want your students to work through during the school year? Are there other key ideas in geometry that you do not see reflected in the curriculum you teach?

J2 *Mini-interview.* Interview a single student about the different kinds of polygons the student knows. Use age-appropriate terms (see web page for suggested prompts). This assignment should serve as a dry run for the student interview assignment, so see that portion of the syllabus for the general format, but keep in mind that this mini-interview should cover only this question and thus be much smaller in scope.

J3 *"What's an angle?".* Ask several students to define or explain what an angle is (use age-appropriate vocabulary). Report their responses verbatim, and then compare them with the meanings for angles identified in the articles by Keiser. Do their definitions cover all the conceptual models?

J4 *Got angle?* In Session 4, we address questions such as, What does it mean for a triangle to "have" 180°? What does it mean for a quadrilateral to have 360°? However, such statements are sometimes made for figures that are not polygons: What does it mean to say that a line has 180°? that a circle has 360°? Summarize your current ideas about these issues.

J5 *Mini-case study.* Pose, to a group of K–8 students, a question similar to those in the cases we have read or seen, or from one of our class discussions. (Make sure it is appropriate to the students' grade level.) Write about your question, what you expected, and what actually happened. Did anything surprise you? Please describe specific examples of what your students say and do. Examining the work of a few students in detail may be more helpful than trying to incorporate the responses of every student. Think of this as a dry run for the case study.

J6 *Triangles interview.* Ask several students to write down answers to the following questions: (1) What is a triangle? (2) Can you draw five different triangles for me? (3) Can you draw 5 other shapes which are not triangles? (4) Why are these other shapes not triangles? Then write a reflection (not a transcription!) on the results, addressing the following issues:

• Did all the triangles have a horizontal "bottom" side, i.e., "sitting up", like this: Δ ?
If so, why do you think so? • At what van Hiele level(s) are the students' answers? Explain. Be sure to give ages or grade levels for the students.

J7 *What does "shape" mean?* First, write a paragraph responding to the following two questions:
(a) What do we mean by the word "shape"? Try to explain without resorting to a list of shapes. (b) What is it that makes two shapes the same or different?

Next, ask several individuals (both children and adults) these same two questions.
Report their answers, and compare them to your original ideas.

J8 *One-problem mini-write-up.* Write a paper detailing either the regular tessellations of the plane or the regular polyhedra. Explain clearly your reasons (a) why those that exist satisfy the definition, and (b) how you know there aren't any others. Consider this a dry run for the two-problem paper: see the corresponding instructions for format and elements, but consider only the part of that assignment dealing with writing up the college-level problem, and remember this journal entry is smaller in scope.

J9 *Curriculum evaluation.* You will conduct an evaluation of the ideas entailed in a single lesson from a given set of mathematics curriculum materials. See DMI handout for full prompt.

J10 *Synthesis.* Reflecting over your thoughts and experiences during the course, write one short paragraph on each of the following (for a total of four): (a) changes in key ideas, relative to those you identified in your first journal assignment of the course; (b) the most striking mathematical idea of the course so far; (c) the most striking idea of the course so far about student learning/thinking; (d) a mathematical idea that still puzzles or intrigues you.

2. Student interview

In order to develop (or strengthen) the habit of attending to student thinking in detail, you will conduct an interview with a student from your class to determine the extent of her/his understanding of a specific mathematical topic. You may choose the student and topic, but the interview should involve a topic from geometry. Begin by obtaining all necessary permissions to conduct and record (audio or video) the interview (this will keep you from needing to make detailed notes during the interview). Explain to all interested parties (including the student!) that you need the student's help for a class in which you are studying how students learn, and that this interview will not affect the student's grades; it will just help you understand better how the student thinks. (Recording the interview will keep you from needing to make detailed notes during the interview.)

Before the interview, you may choose to get a copy of recent written work by the student that shows her/his ability to reason and problem-solve (the work need not be error-free, but the student should have made enough progress that the two of you can discuss the problem). Make sure the student is familiar with the paper, and begin the interview by asking the student to explain her/his work, including what difficulties s/he encountered.

Continue the interview by asking further questions about the mathematical topic involved (I have a separate handout on interviewing tips on the course web site). Remember that *in order to determine the limits of a student's knowledge, you must continue until you reach a question which the student either cannot answer or answers incorrectly for reasons other than a simple careless error.* You should be able to do this without making the student feel badly.

After the interview, use your recording to make a more detailed analysis of the student's thinking, with regard to both problem-solving abilities and knowledge of the particular mathematical topic. Give an overall narration of the interview (e.g., say what specific tasks or problems you asked the student to work on). Use specific details or quotes to support your analysis. Conclude your write-up with an explicit summary of what the student knows, what the student does not know, and what the student is ready (or needs) to work on next (see interview tips handout for more). Identify at which van Hiele level you find the student's thinking, and why.

3. Case study

During the course we will read and discuss in class several case studies, all describing events in other teachers' classrooms. For this assignment, you are to write a short (roughly 3–5 pages) case study describing an episode from your own teaching practice (or observation) driven by student thinking. A case is neither a complete transcript of a lesson nor as prefabricated as an interview, although it is very helpful to include direct quotes and dialogue from students. You must base your case on a conversation for which you were present, and preferably in which you were involved, but it could come out of a lesson you observed, or a conversation among two or more students. You may choose to narrow in on one or two students, or on one small group, or you may describe a whole-class conversation. The most important thing is that the episode illustrate some aspect of children's mathematical thinking, as well as your ability to reflect on that thinking and identify larger teaching issues raised for you by the episode you describe.

In writing your case study, begin by describing briefly the class's larger context (including grade level) and the mathematical topic; then describe the relevant parts of the conversation in as much detail as you can manage. Report direct evidence of student thinking, and then explain what you were thinking as you worked with the students. Finish up by summarizing your evaluation of the students involved and saying what issues and questions you still have after this conversation. Include an analysis of the students' thinking, and questions the case raises for you (including outside the context of this one class). We will discuss the writing of cases in more detail before they are due, but of course you are encouraged to begin sooner, especially if you have a good conversation fresh in your mind. I will be glad to work with you one-on-one in helping you write your case.

Your case study must touch on a mathematical topic involving geometry.

4. 2-problem paper

In order to understand the concepts underlying geometry (including teaching it), you must gain experience in explaining its applications. As a summative evaluation of the mathematical portion of this course, you will submit a paper detailing your mathematical work on a *college-level* problem from this course which you solved completely, and a problem from K–8 mathematics which you believe is related. (Please check with me prior to submission to verify that the problem you have selected as college-level is appropriate and not from K–8 mathematics! Acceptable problems will be listed on the course web page.)

For the college-level problem, give a thorough explanation of the original problem (paraphrased), its context, the strategies you used to approach it, what the solution is (and why! that's the tricky part), and what the solution means in context. Distinguish carefully between conjectures and rigorous arguments. Feel free to use drawings, graphs, diagrams, tables, etc. if necessary (hand-drawn if necessary). A handout on writing mathematics papers is available on the class web page. Cite outside sources clearly.

Also select a problem from K–8 mathematics (possibly, and preferably, from your own classroom) which you believe entails geometric concepts similar to those involved in the college-level problem, and explain the mathematics involved in this problem, clarifying what common ideas the two problems share.

I encourage you to show me a draft of your paper before final submission.

5. Lesson paper

In this course we will study the teaching and learning of ideas related to geometry in K–8 mathematics. As a summative evaluation of the pedagogical aspects of this course, you will develop or select a lesson which fosters the learning of these concepts, teach and document the lesson, and give a short (10-minute) presentation to the class on how it went. The parts of this assignment are as follows:

1. Select or develop a problem intended for use with the students you teach, which involves some aspect of geometry. You may use or adapt a problem from class materials, but be sure it is appropriate for the target audience. (Say where you got it from, and, if you have used it before, in what capacity, and what you learned from it.) The best lessons tend either to integrate multiple strands of mathematics to illustrate connections, or to address significant conceptual issues within a single strand as a summative activity following multiple experiences in developing and exploring a concept.
2. Write a paragraph explaining what geometry concepts are entailed in this problem. (You may use deconstruction if it helps.)
3. Add to the above written descriptions a short sketch of how you plan to use the problem in a lesson, and meet with your instructor to discuss your progress. (This is the lesson draft checkpoint. The above items will also form part of your final paper.)
4. Write a lesson plan that uses the modified problem as a significant problem-solving opportunity with your students. List prerequisite skills.
5. Teach the lesson to your students (see me if this is problematic).
6. Write a one-page reflection on how the lesson went, including what strategies students used to approach the problem, what ideas were raised in its discussion, and to what extent your students' understanding of the underlying geometric concepts—or ability to apply them—changed as a result of the lesson. *Be specific.*
7. Make a one-page handout (you may use front and back if necessary, but it *must* fit on one sheet) summarizing your lesson for the class. Include modified problem, grade level, mathematical topics addressed, and anything your colleagues would need to know in order to use the lesson, including (briefly) any difficulties the students tended to encounter. The handout should *not* be the same as your lesson plan, and must be turned in along with the other documentation at Session 12. Make sure your name is at the top, to facilitate your classmates finding it when you give your presentation.
8. Give a brief (5–10 minutes) presentation to the class on this lesson, using the handout, at our last class meeting. Let me know at least a week ahead of time if you will want A/V equipment (like a projector or document camera) for your presentation.

I encourage you to discuss this project with me as often as you like, throughout the semester. A preliminary draft of the modified problem and lesson idea (not [necessarily] yet taught) is due at Session 10 (see step 3 above). Final documentation is due at Session 12 (so that I can return it to you), including a handout, with the presentations to be given at Session 16.

Bibliography *(TCM=Teaching Children Math., MTMS=Math. Teaching in the Middle School)*

Jeffery J. Boats, Nancy K. Dwyer, Sharon Laing and Mark P. Fratella. Geometric conjectures: the importance of counterexamples, *MTMS* 9(4): 210–215, December 2003.

William M. Carroll. Cross sections of clay solids, *Arithmetic Teacher* 35(7): 6–11, March 1988.

Juli Dixon, Movements in the plane: conjecturing about properties of transformations, *ON-Math* 1(2), Winter 2002.* (* denotes available electronically)

Education Development Center, Inc. (2000). Perspective: What Is Geometry? In *Connected Geometry: A Habits of Mind Approach to Geometry.* Glencoe/McGraw-Hill. pp. 9–11.*

Patrick J. Eggleton. Experiencing radians, *Mathematics Teacher* 92(6): 468–471, September 1999.

[FW] Betz Frederick and Nan Williams. Totally tessellated, *TCM* 10(7): 358, March 2004.

Jane M. Keiser. The role of definition, *MTMS* 5(8): 506–511, April 2000.

[KKF] Jane M. Keiser, Amanda Klee and Karen Fitch. An assessment of students' understanding of angle, *MTMS* 9(2): 116–119, October 2003.

Juan Luis Martínez. The nature of fractals, in *Third Apex to Fractovia*, http://www.fractovia.org/art/what/what_ing1.shtml. 6pp. Last updated 03 June 2006.*

[RLPF] Rheta N. Rubinstein, Glenda Lappan, Elizabeth Phillips, and William Fitzgerald. Angle sense: a valuable connector, *Arithmetic Teacher* 40(6): 352–358, February 1993.

[EFS] Deborah Schifter, Virginia Bastable, and Susan Jo Russell. (2002). *Examining Features of Shape Casebook*. Parsippany, NJ: Dale Seymour/Pearson.

Jessica L. Schoffel and M. Lynn Breyfogle. Reflecting shapes: same or different?, *TCM* 11(7): 378–382, March 2005.

Marjorie Senechal. (1990). Shape. In *On the shoulders of giants: new approaches to numeracy*, ed. Lynn Arthur Steen. Washington, D.C.: National Academy Press. pp. 139-148.*

Janet M. Sharp and Corrine Heimer. What happens to geometry on a sphere?, *MTMS* 8(4): 182–188, December 2002.

Thomas W. Shilgalis and Carol T. Benson. Centroid of a polygon—three views, *Mathematics Teacher* 94(4): 302–307, April 2001.*

William Stegemoller and Rebecca Stegemoller. A path to discovery, *MTMS* 9(8): 458–464, April 2004.

John A. Van de Walle. (2001). Geometric Thinking and Geometric Concepts. In *Elementary and middle school mathematics: teaching developmentally*, 4th ed. Boston: Allyn and Bacon. pp. 306–312.*

Nancy C. Whitman. Experimenting with Crazy Cars, *TCM* 7(2): 70–72, October 2000.

Lynda R. Wiest. Take time for action: investigating students' thinking about nets, *MTMS* 11(3): 140–143, October 2005.

Calendar

A tentative schedule with topics is given below (subject to updating).

Sess.	Date	Topic	Readings/Cases Due	Assignments Due
1	5/26	Dimension	EDC, Van de Walle	J1
2	5/28	Developing geometric language	EFS1	J2*
3	6/02	Developing geometric terms	EFS2, Martínez, TBA	J3*
4	6/08	Notions of angle	EFS3, Stegemollers, Whitman	Interview*
5	6/09	Applications of angle	RLPF, KKF, Eggleton, FW	J4
6	6/10	Definitions in geometry	EFS4, Keiser	J5*
7	6/11	Non-Euclidean geometries	Sharp & Heimer	J6*
8	6/15	Extending & applying definitions	Boats et al.	Case study*
9	6/16	Comparing shapes	EFS5, Senechal	J7*
10	6/17	Similarity & congruence	Schoffel & Breyfogle, Dixon	lesson draft
11	6/18	Symmetry	— (bring Platonic solids)	J8
12	6/22	2-D representations...	EFS6, Carroll	Lesson paper*
13	6/23	...of 3-D objects	Wiest	2-problem paper
14	6/24	Notions of center	EFS7	J9
15	6/25	Synthesis	EFS8, Shilgalis & Benson	J10[, rewrite]
16	6/29	Final presentations	—	Give present'ns

EFSn means Chapter n in the EFS casebook. TBA = to be announced.