Math 5370–001, Fall 2011:
Problem Solving Concepts in K–8 Mathematics

Mondays 5:00–8:00 PM, PKH 103, UTA

Instructor: Dr. Christopher Kribs Zaleta
email: kribs@mathed.uta.edu
Office: 483 Pickard Hall, UTA campus
WWW: http://mathed.uta.edu/kribs/
Phone: (817)272-5513, fax 272-5802
Office Hours: before/after class, Mon. 4 PM & by appt.

Prerequisites: Graduate standing and consent of instructor
Text: Frank K. Lester, Jr. (ed.) (2003). *Teaching mathematics through problem solving: prekindergarten–grade 6.* Reston, VA: NCTM. [referred to in calendar as **TMPS**]
(Supplementary materials will be provided in class or on the course web page.)
Course home page: http://mathed.uta.edu/kribs/5370.html
Last day for withdrawal: November 4

LEARNING OUTCOMES: Students who complete this course will be able to:

- *apply a broad range of problem solving techniques*, as well as know how and when to apply them
- *explain the roles of problem solving and disposition* in the learning of mathematics
- *explain the nature of problem solving* as an instructional strategy, a curricular goal, and more process than product
- *explain the roles of technology and of conjecture & proof* in problem solving
- *describe and communicate* mathematical ideas
- *understand and analyze* the mathematical thinking of others (especially students)
- *develop all of the above in K–8 students*, i.e., to help them become mathematical problem solvers as well
- *unpack, modify and develop rich mathematical problems*
- *assess problem solving performance*

FORMAT: This course will study problem solving in several ways: through work on challenging mathematical problems to develop our own problem solving abilities (and communicating the results, to develop our expository abilities); discussing what research has discovered about the development of problem solving abilities at the K–8 levels; examining specific instances of K–8 students' attempts to solve problems, through both case studies and our own classroom practice; considering in detail how the features of mathematical problems affect the ways they can be approached, and how to modify problems to provide rich problem-solving opportunities for our students; and studying how the problem-solving process forms the core of mathematics pedagogy.

Before class each week you will read a single research paper (or part of one) and a case study from K–8 mathematics, and make notes on them in preparation for class discussions of them. You will also often work on mathematics problems outside of class, to facilitate their discussion in class. We will typically begin class by working on new mathematics problems and discussing their solutions, in both small and large groups. We will follow this up by discussing the assigned readings, as well as other topics related to problem solving. We will typically end class with time for reflection on how the topics we have discussed apply to our own classrooms. During class discussions we will often refer back to work we have done earlier in the course, so please bring your notes and papers from previous sessions to class.

POLICIES:

• Students who are not classroom teachers will need to make arrangements to interact with K–8 students for many of the assignments (those starred * on the calendar).

• Students are expected to be on time, prepared and ready to work every week. This class meets every Monday from 5:00 PM to 8:00 PM from August 29 to December 12. Each student is allowed the equivalent of one week's absence (3 total hours) for whatever reason without penalty. All subsequent absences (including arriving significantly late) will result in the reduction of the final course grade by one-half letter grade (5%) for each absence. *Students who miss an entire class must talk with the instructor prior to the next class.*

• With the exception of diagrams and examples of student work, written assignments are expected to be typed and use correct grammar and punctuation.

• Each student is allowed one late submission during the semester. The paper must be submitted before the beginning of the class period following that in which it was due. Papers not submitted by the end of class time on the due date are considered late. Submission of a late paper constitutes the student's agreement that this is the one allowed late assignment.

• Each student is allowed one electronic submission during the semester. Electronic submissions must be complete and not missing any ancillary materials such as student work necessary for grading. (If the electronic submission is made late, then it is both the only late paper allowed and the only electronic submission allowed.) This does not include drafts sent for consultation prior to submission, but consultation must take place in person or via telephone.

• Each student is allowed to submit one revised paper for a regrade, under the following terms: The revised paper and the graded original must be turned in together at the penultimate class meeting. The new grade replaces the original. Students are encouraged to consult with the instructor prior to submitting a revised paper.

• As a sign of respect for your peers and our common work, please keep all cellular phones, computers, and other electronic devices turned off during class. In emergencies cell phones may be set to vibrate only, and brief calls taken in the hallway outside.

• This course follows University policies on topics such as drops, withdrawals, academic integrity, accommodating disabilities, etc. Please see the attached supplement for further details.

GRADES: Your grade for the course will be determined by five elements, each of which has equal weight: (1) preparation and participation, (2) a written student interview, (3) a short case study, (4) a paper detailing your own mathematical work, and (5) a lesson involving a problem you modify to provide problem solving opportunities for your students. All of these are detailed below.

Assignments

1. Journal

On days in which there is not a major assignment due, you will write a short (about one page) reflection in response to a prompt given below. Many of the prompts are also given in the DMI handouts distributed in class. Some involve "action research" reports in which you will write about your own students' mathematical work. You will often use and discuss your responses in class, within your small groups and in large group. These reflections are to be turned in at the end of class; I will respond to them in writing and return them at our next class meeting. Grading will be limited to verifying that responses are appropriate in topic and scope (length).

Your journal entries will serve to document your preparation for class each day (and your growth over time); your preparation and participation grade will be based half on your journal (entries should be complete each day before class) and half on your participation in class discussions (I expect participation in large-group discussion at least ten of the fifteen times we will meet).

J1 *What is problem solving?.* What did the term "problem solving" mean to you a year ago? What are the key issues that you see entailed in this topic? Be as specific as you can.

J2 *Mini-interview.* Interview a single student about her/his knowledge of problem solving techniques (or, alternatively, her/his awareness of different ways to solve a single specific problem recently solved in class). Use age-appropriate terms. This assignment should serve as a dry run for the student interview assignment, so see that portion of the syllabus for the general format, but keep in mind that this mini-interview should cover only one question and thus be much smaller in scope.

J3 *Student problem solving through representations.* Describe a specific instance from your classroom practice in which one or more of your students made use of representation as a problem-solving tool: that is, in which the students' choice of representation (not just clear notation or reasoning using the teacher's representation) facilitated the solution. Give the problem and explain how (at least two) different representations shape the corresponding solutions (not necessarily the answers).

J4 *Mini-case study.* Pose, to a group of K–8 students, a single question similar to those in the cases we have read or seen, or from one of our class discussions. (Make sure it is appropriate to the students' grade level.) Write about your question, what you expected, and what actually happened. Did anything surprise you? Please describe specific examples of what your students say and do. Examining the work of a few students in detail may be more helpful than trying to incorporate the responses of every student. Think of this as a dry run for the case study.

J5 *Problems in need of enrichment.* Select three problems from the mathematics curriculum for your grade which involve important topics but provide your students little or no opportunity for problem solving. (They can come from your text, district materials, etc. but should be problems you might be expected to use in class.) Explain (separately) why you chose each one, and say what you might like to arise in a class discussion of a "richer" version of each one. (That is, do not actually try to rewrite the problems; just say as specifically as possible what you wish each one could accomplish.)

J6 *Deconstructing pentominoes.* (i) Write a formal deconstruction of the pentominoes problem, and (ii) (re)structure it at least three different ways (that is, change the presentation, not the problem itself).

J7 *Student generalization.* Introduce "diffy boxes" (see coursepack for details) to a class of students, using a few student-generated examples, and then have them work—first individually and then in small groups—to try to find any patterns or generalizations that occur to them. Collect their written work and write a summary of what students discussed verbally (to supplement the written work). Then use this data to write a one-page analysis of what kinds of generalizations the students investigated. Include copies (or scans) of student work (names removed) and initial summary.

J8 *One-problem mini-write-up.* Write a paper detailing the solution to either Squares problem 1 or Squares problem 2. Consider this a dry run for the problem-solving paper: see the corresponding instructions for format and elements, but consider only the first part of that assignment, and remember this journal entry is smaller in scope.

J9 *Assessing problem solving.* The case study "Right or Wrong" in this week's readings involves the grading of two student papers to the same problem. Begin by reading only the first page, including the papers by Chris and Pat. Develop an explicit 5-point grading scale for scoring responses to this specific problem (not a generic rubric), and write a paragraph explaining what you think the most important issues involved in this problem are. Then apply your scale to both papers, and write a short paragraph explaining why each paper received the grade (0–5) you assigned it. Finally, read the rest of the case study, and write a fourth paragraph in which you respond either to the issues raised in the last page of the study, or to the scores the teachers in the study gave these papers. Be ready to discuss your responses in class.

J10 *Synthesis.* Reflecting over your thoughts and experiences during the course, write one short paragraph on each of the following (for a total of four): (a) changes or developments in what you see as key ideas in problem solving, relative to those you identified in your first journal assignment of the course; (b) the most striking *mathematical* idea of the course so far; (c) the most striking idea of the course so far about student learning/thinking; (d) a mathematical idea that still puzzles or intrigues you.

2. Student interview

In order to develop (or strengthen) the habit of attending to student thinking in detail, you will conduct an interview with a student from your class to assess her/his understanding of a specific mathematical topic. You may choose the student and topic, but the interview should involve a topic with which the student has recently had some problem-solving opportunities. Begin by obtaining all necessary permissions to conduct and record (audio or video) the interview; explain to all interested parties (including the student!) that you need the student's help for a class in which you are studying how students learn, and that this interview will not affect the student's grades; it will just help you understand how s/he thinks. (Recording the interview will keep you from needing to make detailed notes during it.)

Before the interview, get a copy of recent written work by the student showing her/his ability to reason and problem-solve (the work need not be error-free, but there should be enough progress made to discuss the problem). Make sure the student is familiar with the paper, and begin the interview by asking him/her to explain the work, including what difficulties s/he encountered.

Continue the interview by asking further questions about the mathematical topic involved (see the handout on interviewing tips on the course web site). You will need to use both pre-prepared questions and ad hoc follow-up questions to develop a coherent line of questioning. Remember that *in order to determine the limits of a student's knowledge, you must continue until you reach a question which the student either cannot answer or answers incorrectly for reasons other than a simple careless error*. You should be able to do this without making the student feel badly.

After the interview, use your recording to make a more detailed analysis of the student's thinking, with regard to both problem-solving abilities and knowledge of the particular mathematical topic. Begin with a brief introduction to provide context. Give an overall narration of the interview (e.g., say what specific tasks or problems you asked the student to work on). Use specific details or quotes to support your analysis. Conclude your write-up with an explicit summary of what the student knows, what the student does not know, and what the student is ready (or needs) to work on next (see interview tips handout for more).

3. Case study

During the course we will read and discuss in class several case studies, all describing events in other teachers' classrooms. For this assignment, you are to write a short (roughly 3–5 pages) case study describing a mathematical discussion involving one or more students, similar to these cases. A case is neither a complete transcript of a lesson nor as prefabricated as an interview, although it is very helpful to include direct quotes and dialogue from students.

You must base your case on a conversation for which you were present, and preferably in which you were involved, but it could come out of a lesson you observed, or a conversation among two or more students. You may choose to narrow in on one or two students, or on one small group, or you may describe a whole-class conversation. The most important thing is that the episode illustrate some aspect of children's mathematical thinking. It must also center on the development of problem-solving abilities.

In writing your case study, begin by describing briefly the class's larger context (including grade level) and the mathematical topic; then describe the relevant parts of the conversation in as much detail as you can manage. Include what you are thinking as you work with the students. Finish up by summarizing your evaluation of the students involved and saying what issues and questions you still have after this conversation. Include an analysis of the students' thinking, and questions the case raises for you. *It is important that your reflection address teaching issues beyond the one topic and set of students involved*, in order to document your ability as a reflective practitioner to make connections that inform your teaching practice more broadly.

We will discuss the writing of cases in more detail before they are due, but you are encouraged to begin sooner if you have a good conversation fresh in your mind. I will be glad to help you.

4. 2-problem paper

In order to understand the various processes involved in problem solving (including teaching it), you must gain experience in problem solving yourself. As a summative evaluation of the mathematical portion of this course, you will submit a paper detailing your mathematical work on two *college-level* problems from this course: one which you solved completely, and one which you did not. You *must* check the list on the course web page or meet with me individually to approve and verify the problem you wish to write up.

For the problem which you solved completely, give a thorough explanation of the original problem (paraphrased), its context, the strategies you used to approach it, what the solution is (and why! that's the tricky part), and what the solution means in context. Distinguish carefully between conjectures and rigorous arguments. Feel free to use drawings, graphs, diagrams, tables, etc. if necessary.

Also select a problem you have worked on in this course which you have not yet been able to solve, but feel you would eventually be able to solve. Describe the problem, describe what you have attempted, and reflect on (describe) what you may have learned from working on the problem so far (if anything – if nothing, explain why you feel your time working on the problem was wasted). If you think you know the answer, or part of an answer, but cannot prove it, include your conjecture, and clarify its status as conjecture. Regardless, write a brief paragraph on the role of conjecture in problem solving, in regard to either this problem, or learning mathematics more generally.

I encourage you to show me a draft of your paper before final submission.

5. Lesson paper

In this course we will study how to identify and modify the basic elements of a mathematical problem. As a summative evaluation of the pedagogical aspects of this course, you will apply these techniques to one particular problem, develop and teach a lesson based on your modified problem, and give a short (5-minute) presentation to the class on it. The lesson draft checkpoint includes items 1–5 below. The final lesson paper you submit must include *all* of the following components:

1. Select a problem that is intended for use with the students you teach, which you believe as given does not provide real opportunities for problem solving, but could. (Say where you got it from, and, if you have used it before, in what capacity, and what you learned from it.)
2. Unpack, or deconstruct, the problem, by identifying all the basic elements within it (remembering, for instance, that the particular context or numbers used may affect likely strategies), as discussed in class beginning Week 7.
3. Use the rescaling and enrichment techniques discussed in class beginning Weeks 8 and 9 to modify the problem so that it provides significant opportunities for problem solving, while remaining appropriate for your students.
4. Write an analysis of the modified problem by (a) deconstructing it and comparing its elements with those of the original problem, and (b) anticipating what strategies can be used to approach it, including mappings between these strategies, and connections to other areas of mathematics.
5. Submit a one-page draft containing a paragraph for each of the above elements, plus a fifth which sketches briefly how you plan to use the problem in a lesson. (This is the lesson draft checkpoint. The draft [revised if necessary] will also form part of your final paper.)
6. Write a lesson plan that uses the modified problem as a significant problem-solving opportunity with your students. Specify prerequisite knowledge, closure activities, and important discussion points.
7. Teach the lesson to your students (see me if this is problematic). Then write a one-page reflection on how the lesson went, including what strategies students used to approach the problem, what ideas were raised in its discussion, and to what extent your students' problem-solving abilities changed as a result of the lesson. Be specific.
8. Make a one-page handout (you may use front and back if necessary, but it *must* fit on one sheet) summarizing your lesson for the class. Include your name, the problem, grade level, mathematical topics addressed, and anything your colleagues would need to know in order to use the lesson, including (briefly) any difficulties the students tended to encounter. The handout should *not* be the same as your lesson plan (select details!), and must be turned in with the main paper.
9. Give a brief (5-minute) presentation to the class on this lesson, using the handout, at our last class meeting.

I encourage you to discuss this project with me as often as you like, throughout the semester. A preliminary draft of the selected problem and lesson idea (not [necessarily] yet taught) is due at Session 9 (see step 5 above). Final documentation is due at Session 12, including a handout, with the presentations to be given at Session 16.

Bibliography

Richard Caulfield, Shelly Sheats Harkness, and Robert Riley (2003). Surprise! Turn routine problems into worthwhile tasks, *Mathematics Teaching in the Middle School* 9(4): 198–201.

Ray Hembree and Harold Marsh (1993). Problem solving in early childhood: building foundations. In Robert J. Jensen (ed.), *Research Ideas for the Classroom: Early Childhood Mathematics* (pp. 151-170). New York: MacMillan. National Council of Teachers of Mathematics Research Interpretation Project.

[TMPS] Frank K. Lester, Jr. (ed.) (2003). *Teaching mathematics through problem solving: prekindergarten–grade 6.* Reston, VA: NCTM.

Frank K. Lester and Diana Lambdin Kroll (1990). Assessing student growth in mathematical problem solving. In G. Kulm (ed.), *Assessing higher order thinking in school mathematics* (pp. 53–70). Washington, DC: American Association for the Advancement of Science.

[PSSM] National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

Pólya, G. (1945; 2nd edition 1957). *How to Solve It.* Princeton, NJ: Princeton University Press.

Mary Kay Stein, Margaret Schwan Smith, Marjorie A. Henningsen, and Edward A. Silver (1999). *Implementing standards-based mathematics instruction: a casebook for professional development.* New York: Teachers College Press.

Calendar

A tentative schedule with topics is given below (subject to updating).

Date	Wk	Topic	Readings	Assignments Due
29 Aug	1	The nature of PS	SC1* (FDRP 119)	*none*
05 Sep†	*11*	Mathematical habits	TMPS2, SC 11 (EFS33)	J1
12 Sep	2	PS as process	TMPS7, SC2 (MMO21)	J2*
19 Sep	3	Anatomy of a problem	TMPS9, SC3* (DE 4–5)	J3*
26 Sep	4	Representations in PS	PSSM Repr'ns, SC4 (PFC7)	Student interview*
03 Oct	5	PS techniques	TMPS11, SC5	J4*
10 Oct	6	Patterns & induction	PSSM Prob. Solv., SC6 (PFC12)	J5
17 Oct	7	Deconstructing problems	Stein et al. Ch. 1,2, 10	Case study*
24 Oct	8	Modifying problems	Caulfield et al., SC8*	J6
31 Oct	9	Enriching problems	TMPS5, SC9*	J7*, lesson draft
07 Nov	10	Conjecture & proof	PSSM Rsng & Proof, SC10 (EFS35)	J8
14 Nov	12	Assessment	Lester & Lambdin Kroll, SC12	J9
21 Nov	13	PS as pedagogy	Hembree & Marsh, SC 13 (FDRP 62)	Lesson paper*
28 Nov	14	Technology in PS	TMPS13, SC 14* (DE 6–8)	Problem-solving paper
05 Dec	15	Linear modeling	TMPS15, SC 15 (PFC14)	J10[, rewrite]
12 Dec	16	Synthesis	*none*	Presentations

* denotes readings available online or assignments requiring interaction with K–8 students.
SCn refers to Supplementary Case n (information in parentheses identifies the source for some).
TMPSn refers to Chapter n in TMPS.

See bibliography for further details of readings (nonelectronic readings are available at the UTA Libraries).

† Since September 5 is Labor Day, a holiday, our class will have a virtual meeting that week. Instructions will be posted on the course home page as well as distributed at the first class meeting.