

# Math 5377-001, Summer I 2015:

## Algebraic Reasoning in K–8 Mathematics

selected days & times, PKH 311

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*Office Hours:* before/after class and by appointment

*Prerequisites:* MATH 5375

*Text materials:* (1) DMI's *Reasoning Algebraically about Operations* casebook [RAO],  
(2) *Thinking Mathematically* by Carpenter et al. [TM]. See bibliography for full citations;  
additional materials will be provided in class or on the course web page.

*Course home page:* <http://mathed.uta.edu/kribs/5377.html>

*Last day for withdrawal:* June 29

Class policy on drops, withdrawals, academic honesty, and disabilities follows the University policy on these matters (see syllabus addendum). Plagiarized papers will earn a grade of 0.

**LEARNING OUTCOMES:** The successful student will be able to:

- *identify algebraic ideas* present at each level in the K–8 curriculum and foster their age-appropriate learning by students
- *understand, use, and analyze appropriate representations* of algebraic ideas (generalizations) at the concrete, semi-concrete, and abstract levels
- *understand, use, and analyze properties of operations and relations*
- *recognize and identify different stages of learners' notions of generalizations and proof*
- *describe and communicate* mathematical ideas
- *understand and analyze* the mathematical thinking of others (especially students)

**FORMAT:** This course will study algebraic reasoning in several ways: through work on challenging mathematical problems to develop our own algebraic reasoning abilities (and communicating the results, to develop our expository abilities); discussing what research has discovered about the development of algebraic reasoning abilities at the K–8 levels; and examining specific instances of K–8 students' algebraic reasoning, through both case studies and our own classroom practice.

Before class each week you will read professional articles and/or case studies from K–8 mathematics, and make notes on them in preparation for class discussions. You will also often work on mathematics problems outside of class, to facilitate their discussion in class. We will typically begin class by working on new mathematics problems and discussing their solutions, in both small and large groups. We will follow this up by discussing the assigned readings, as well as other related topics. We will typically end class with time for reflection on how the topics we have discussed apply to our own classrooms. During class discussions we will often refer back to work we have done earlier in the course, as well as in the prerequisite course (MATH 5375/WNO), so please bring your notes and papers from previous sessions to class.

**POLICIES:**

- Students who are not classroom teachers will need to make arrangements to interact with K–8 students for many of the assignments (those starred \* on the calendar).
- Students are expected to be on time and ready to work every week. This class meets Tue–Thu 5–8 PM through 6/11 and 3:30–6:30 PM 6/15–6/25. Each student is allowed the equivalent of one session's absence (3 total hours) for any reason without penalty. All subsequent absences

(including arriving significantly late) will reduce the final course grade by one-half letter grade (5%) per absence. See DMI handout for policy on making up work from a missed class.

- Except for examples of student work, diagrams, equations, etc., written assignments are expected to be typed and use correct grammar and punctuation.

- Each student is allowed one late submission during the semester. The paper must be submitted before the beginning of the class period following that in which it was due. Papers not submitted by the end of class time on the due date are considered late. Submission of a late paper constitutes the student's agreement that this is the one allowed late assignment.

- Each student is allowed one electronic submission during the semester. Electronic submissions must be complete and not missing any ancillary materials such as student work necessary for grading. (If the electronic submission is made late, then it is both the only late paper allowed and the only electronic submission allowed.) This does not include drafts sent for consultation prior to submission, but consultation *must* take place in person or via telephone.

- Each student is allowed to submit one revised paper for a regrade, under the following terms: The revised paper and the graded original must be turned in together at the penultimate class meeting. The new grade replaces the original. Students should consult with the instructor before submitting a revised paper.

**GRADES:** Your grade for the course will be determined by five elements, each of which has equal weight: (1) journal entries and participation, (2) a written student interview, (3) a short case study, (4) a paper detailing your own mathematical work, and (5) a lesson involving a problem you select to develop algebraic reasoning in your students. All of these are detailed in the next section.

## Assignments

### 1. Journal

On most days in which there is not a major assignment due, you will write a short (about one page) reflection in response to a prompt given below. Many of the prompts are also given in the DMI handouts distributed in class. Some involve “action research” reports in which you will write about your own students’ algebraic reasoning. You will often use and discuss your responses in class, within your small groups and in large group. These reflections are to be turned in at the end of class; I will respond to them in writing and return them at our next class meeting. Grading will be limited to verifying that responses are appropriate in topic and scope (length).

Your journal entries will serve to document your preparation for class each day (and your growth over time); your preparation and participation grade will be based half on your journal (entries should be complete each week before class) and half on your participation in class discussions (I expect participation in large-group discussion at least ten of the fifteen times we will meet, and professional behavior throughout).

J1 *Key ideas.* What are the key ideas of algebraic reasoning for you? Consider the following:

- When you hear/see the word *algebra*, what mathematical ideas come to your mind?
- What, if anything, does algebra have to do with the content you teach?
- What might it mean to engage with children in discussing algebraic ideas?
- What do you see as the key elements of algebraic reasoning?

J2 *Mini-case study.* Ask a group of K–8 students to define an even number. Note what each student said, and compare their responses to the definitions we have considered in class. Did anything surprise you? Be specific in describing what the students say and do. Examining the work of a few students in detail may be more helpful than trying to incorporate the responses of an entire class. Think of this as a dry run for the case study. (can also use Guess My Rule)

- J3 *Articulating generalizations.* Students and teachers alike may struggle to articulate the general mathematical ideas they discuss. In preparation for class discussion, write out in words and/or symbols the generalizations present in the student work in RAO Chapter 2.
- J4 *Mini-interview.* Interview a single student to see to what extent (s)he can articulate generalizations about the result of adding two odd numbers. This assignment should serve as a dry run for the student interview assignment, so see that portion of the syllabus for the general format, but keep in mind that this mini-interview should cover only this question and thus be much smaller in scope.
- J5 *Work in progress.* This reflection is about the mathematics *you* are learning in this course, not about your students' learning. (a) Which of the mathematical issues discussed in this course make sense to you in new ways now? Explain. (b) Which ideas are you still working on? How are you working on them?
- J6 *Representations, and the "Laws of Arithmetic".* (a) At this point in the course, what role(s) do you see the various representations we have explored as having in the development of algebraic reasoning? How do you see the representations as being connected?  
(b) Symbolic representations are a powerful way of communicating ideas concisely, but by the same token they require unpacking to use. Respond to the DMI handout "The Laws of Arithmetic" listing the field properties.
- J7 *One-problem mini-write-up.* Write a paper which proves the commutativity of clock addition four different ways, using each of the representations discussed in class (verbal, symbolic, diagrammatic and tabular). Note that the form of your argument will vary with each representation. Consider this a dry run for the two-problem paper: see the corresponding instructions for format and elements, but consider only the part of that assignment dealing with writing up the college-level problem, and remember this journal entry is smaller in scope.
- J8 *Relational thinking.* First, ask a group of students what the  $=$  sign means, and then ask if they agree with statements like those discussed in the readings (e.g.,  $10=3+7$ ,  $5+5=3+7$ ). Are their responses to the second set of prompts consistent with the definition they gave? Second, reflect on how you yourself use the  $=$  sign. Finally, how does teaching practice have to change to foster relational thinking rather than operational thinking (with regard to  $=$ )?
- J9 *Operations and functions.* At the beginning of this course we noted that operations and functions are really the same kind of mathematical object, but we treat them quite differently; indeed, operations seem to belong to elementary grades, while function concepts are one of the main precursors to calculus. Informed by your experiences (recent and not), compare **directly** your own understandings of "operation" and "function".
- J10 *Synthesis.* Based on the writing work done in class, write a short paragraph responding to each of the following questions:
- What does "algebra" or algebraic reasoning mean to you now?
  - Compare your response to the previous question with your response to the same question in Journal 1. How have your ideas changed or developed?
  - What ideas related to teaching algebraic thinking have changed for you? Describe some specific teacher moves you believe are necessary or helpful in teaching algebraic reasoning.
  - What does it mean to state a mathematical generalization in a K–8 classroom?
  - What does it mean to construct a mathematical proof (argument) in a K–8 classroom?
  - What are the connections among contextual, concrete, pictorial, and abstract representations of algebraic concepts?

## 2. Student interview

In order to develop (or strengthen) the habit of attending to student thinking in detail, you will conduct an interview with a student from your class to assess her/his understanding of a specific mathematical topic. You may choose the student and topic, but the interview should involve a major topic from this course. Begin by obtaining all necessary permissions to conduct and record (audio or video) the interview; explain to all interested parties (including the student!) that you need the student's help for a class in which you are studying how students learn, and that this interview will not affect the student's grades; it will just help you understand how the student thinks. (Recording the interview will keep you from needing to make detailed notes during it.)

Before the interview, get a copy of recent written work by the student showing her/his ability to reason and problem-solve (the work need not be error-free, but there should be enough progress made to discuss the problem). Make sure the student is familiar with the paper, and begin the interview by asking him/her to explain the work, including what difficulties s/he encountered.

Continue the interview by asking further questions about the mathematical topic involved (see the handout on interviewing tips on the course web site). You will need to use both pre-prepared questions and ad hoc follow-up questions to develop a coherent line of questioning. Remember that *in order to determine the limits of a student's knowledge, you must continue until you reach a question which the student either cannot answer or answers incorrectly for reasons other than a simple careless error*. You should be able to do this without making the student feel badly.

After the interview, use your recording to make a more detailed analysis of the student's thinking, with regard to both problem-solving abilities and knowledge of the particular mathematical topic. Begin with a brief introduction to provide context. Give an overall narration of the interview (e.g., say what specific tasks or problems you asked the student to work on). Use specific details or quotes to support your analysis. Conclude your write-up with an explicit summary of what the student knows, what the student does not know, and what the student is ready (or needs) to work on next (see interview tips handout for more).

## 3. Case study

During the course we will read and discuss in class several case studies, all describing events in other teachers' classrooms. For this assignment, you are to write a short (roughly 3–5 pages) case study describing a mathematical discussion involving one or more students, similar to these cases. A case is neither a complete transcript of a lesson nor as prefabricated as an interview, although it is very helpful to include direct quotes and dialogue from students.

You must base your case on a conversation for which you were present, and preferably in which you were involved, but it could come out of a lesson you observed, or a conversation among two or more students. You may choose to narrow in on one or two students, or on one small group, or you may describe a whole-class conversation. The most important thing is that the episode illustrate some aspect of children's mathematical thinking. It must also center on a mathematical topic involving algebraic reasoning.

In writing your case study, begin by describing briefly the class's larger context (including grade level) and the mathematical topic; then describe the relevant parts of the conversation in as much detail as you can manage. Include what you are thinking as you work with the students. Finish up by summarizing your evaluation of the students involved and saying what issues and questions you still have after this conversation. Include an analysis of the students' thinking, and questions the case raises for you. *It is important that your reflection address teaching issues beyond the one topic and set of students involved*, in order to document your ability as a reflective practitioner to make connections that inform your teaching practice more broadly.

We will discuss the writing of cases in more detail before they are due, but you are encouraged to begin sooner if you have a good conversation fresh in your mind. I will be glad to help you.

## 4. 2-problem paper

In order to understand the concepts underlying algebraic reasoning (including teaching it), you must gain experience in explaining its applications. As a summative evaluation of the mathematical portion of this course, you will submit a paper detailing your mathematical work on a *college-level* problem from this course which you solved completely, and a problem from K–8 mathematics which you believe is related. You *must* check the course web page or meet with me individually to approve and verify the problem you wish to write up.

For the college-level problem, give a thorough explanation of the original problem (paraphrased), its context, the strategies you used to approach it, what the solution is (and why! that’s the tricky part), and what the solution means in context. Distinguish carefully between conjectures and rigorous arguments. Feel free to use drawings, graphs, diagrams, tables, etc. if necessary.

Also select a single problem (*not* a lesson) from K–8 mathematics (possibly multi-part, and preferably from your own classroom) which you believe entails algebraic reasoning concepts similar to those involved in the college-level problem, and explain in detail what those concepts are, clarifying in a paragraph what common ideas the two problems share. Include the prompt in your paper. Limit your analysis to properties of the problems themselves; do not focus on student work (unlike in the other major papers).

I encourage you to show me a draft of your paper before final submission.

## 5. Lesson paper

In this course we will study the teaching and learning of ideas related to algebraic reasoning in K–8 mathematics. As a summative evaluation of the pedagogical aspects of this course, you will develop or select an exemplary lesson which fosters learning these concepts, teach and document the lesson, and give a short (10-minute) presentation to the class on it. The lesson draft checkpoint includes items 1–3 below. The final lesson paper you submit must include *all* of the following components:

1. Select or develop a problem intended for use with the students you teach, which involves some aspect of algebraic reasoning. You may use or adapt a problem from class materials, but be sure it is appropriate for the target audience. (Say where you got it from, and, if you have used it before, in what capacity, and what you learned from it.) The best lessons tend either to integrate multiple strands of mathematics to illustrate connections, or to address significant conceptual issues within a single strand as a summative activity following multiple experiences in developing and exploring a concept. Specify prerequisite knowledge.
2. Write a paragraph explaining what concepts from this course are entailed in this problem. (You may use deconstruction if it helps you identify them, but write in paragraph form.)
3. Add to the above written descriptions a short sketch of how you plan to use the problem in a lesson, and meet with your instructor to discuss your progress. (This is the lesson draft checkpoint. The above items will also form part of your final paper.)
4. Write a lesson plan that uses the problem/activity as a significant problem-solving opportunity with your students. Include closure activities, and important discussion points.
5. Teach the lesson to your students (see me if this is problematic). Then write a one-page reflection on how the lesson went, including what strategies students used to approach the problem, what ideas were raised in its discussion, and to what extent your students’ understanding of the underlying algebraic thinking concepts—or ability to apply them—changed as a result of the lesson. Be specific.
6. Make a one-page handout (you may use front and back if necessary, but it *must* fit on one sheet) summarizing your lesson for the class. Include the problem, grade level, mathematical topics addressed, and anything your colleagues would need to know in order to use the lesson, including (briefly) any difficulties the students tended to encounter. The handout should *not* be the same as your lesson plan (select details!), and must be turned in with the main paper.
7. Give a brief (10-minute) presentation to the class on this lesson, using the handout, at our last class meeting.

I encourage you to discuss this project with me as often as you like, throughout the semester. A preliminary draft of the selected problem and lesson idea (not [necessarily] yet taught) is due at Session 8 (see step 3 above). Final documentation is due at Session 11 (so that I can return it to you), *including a handout*, with the presentations to be given at Session 16.

## Bibliography

- Maria L. Blanton, *Algebra and the elementary classroom*. Portsmouth, NH: Heinemann. Chapter 6. **online**
- Maria L. Blanton and James J. Kaput, Developing Elementary Teachers' "Algebra Eyes and Ears", *TCM* 10(2): 70–77, Oct 2003.
- Thomas P. Carpenter, Megan Loef Franke and Linda Levi, *Thinking Mathematically: Integrating Arithmetic & Algebra in Elementary School*. Portsmouth, NH: Heinemann, 2003.
- Michaele F. Chappell, Preparing students to enter the gate, *TCM* 3(6): 266–267, Feb 1997.
- Frances R. Curcio and Sydney L. Schwartz, What does algebraic thinking look like and sound like with preprimary children?, *TCM* 3(6): 296–300, Feb 1997.
- M. Driscoll and J. Moyer, Using students' work as a lens on algebraic thinking, *MTMS* 6(5): 282–287, 2001.
- J. Ferrini-Mundy, G. Lappan and E. Phillips, Experiences with patterning, *TCM* 3(6): 282–288, Feb 1997.
- Alfinio Flores, Subtraction of positive and negative numbers, *MTMS* 14(1): 21–23, Aug 2008.
- Catherine Twomey Fosnot, Young mathematicians at work constructing algebra: an update from Math in the City, *Intersection* 6–7, Spring 2007.
- Kristen Herbert and R. H. Brown, Patterns as tools for algebraic reasoning, *TCM* 3(6): 340–345, Feb 1997.
- Karen Koellner, Mary Pittman, and Jeffrey Frykholm, Talking generally or generally talking in an algebra classroom, *MTMS* 14(5): 304–310, Dec 2008/Jan 2009.
- National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: Author. (Algebra standard overview, pp. 37–40.)
- Deborah Schifter, Virginia Bastable, Stephen Monk, Susan Jo Russell, Elham Kazemi. *Developing Mathematical Ideas Casebook. Number and Operations, Part 3: Reasoning Algebraically about Operations*. Parsippany, NJ: Dale Seymour Publications, 2006.
- Stephen S. Willoughby, Functions from kindergarten through sixth grade, *TCM* 3(6): 314–318, Feb 1997.

## Calendar

A tentative schedule with topics is given below (subject to updating).

Date	Wk	Topic	Readings/Cases Due	Homework Due
20 May	1	Intro. to alg. reasoning	<i>none</i>	J1
27 May	2	Odd and even numbers	RAO1, Chappell, NCTM	J2*
21 May	3	Gen'l'z'ns in add. & sub.	RAO2, Fosnot	J3
26 May	4	Commutativity	RAO3	J4*
28 May	5	Integers	RAO4, Flores	Case study*
09 Jun	6	Identity and inverses	RAO5	Student interview*
10 Jun	7	Distributivity	RAO6	Lesson draft
11 Jun	8	Factor rules & associativity	RAO7, TM9	J5
15 Jun	9	Clock arithmetic	TM pp. v–xiii, 1–7; Blanton pp. 91–102	J6
16 Jun	10	Relations	TM2, Curcio & Schwartz	J7
17 Jun	11	Equality and equivalence	TM3, Blanton pp. 103–120	Lesson paper*
18 Jun	12	Generalizing from patterns	TM4, Ferrini-Mundy et al.	J8*
22 Jun	13	Unknowns and variables	TM5,6, Bl. & Kaput, Koellner et al.	2-problem paper
23 Jun	14	Functions	TM7,8, Willoughby	J9
24 Jun	15	Generalizations & justifications	RAO8, TM10, Driscoll & Moyer, Herbert & Brown	(Rewrites)
25 Jun	16	Synthesis	<i>none</i>	Present'ns, J10

See bibliography for further details of readings (nonelectronic readings are available at the UTA Libraries).

**Note on study time:** Summer courses take a sixteen-week semester and compress it into five weeks. That's a compression factor of more than three! Not only does this time compression leave students less time to "unpack" and reflect between class meetings, but it also means that in order to engage fully in the course, one has to spend more than five times as many hours per day outside of class on coursework as would be true during a long semester. In particular, instead of the usual rule of thumb of six hours per week outside of class for every three hours per week spent in class, the proportion becomes 18 hours per week out of class for 9 hours per week in class. This is equivalent to a half-time job. Please be careful to plan accordingly.

## University Policies

**Drop Policy:** Students may drop or swap (adding and dropping a class concurrently) classes through self-service in MyMav from the beginning of the registration period through the late registration period. After the late registration period, students must see their academic advisor to drop a class or withdraw. Undeclared students must see an advisor in the University Advising Center. Drops can continue through a point two-thirds of the way through the term or session. It is the student's responsibility to officially withdraw if they do not plan to attend after registering. **Students will not be automatically dropped for non-attendance.** Repayment of certain types of financial aid administered through the University may be required as the result of dropping classes or withdrawing. For more information, contact the Office of Financial Aid and Scholarships (<http://wwwb.uta.edu/ses/fao>).

**Americans with Disabilities Act:** The University of Texas at Arlington is on record as being committed to both the spirit and letter of all federal equal opportunity legislation, including the *Americans with Disabilities Act (ADA)*. All instructors at UT Arlington are required by law to provide "reasonable accommodations" to students with disabilities, so as not to dis-criminate on the basis of that disability. Any student requiring an accommodation for this course must provide the instructor with official documentation in the form of a letter certified by the staff in the Office for Students with Disabilities, University Hall 102. Only those students who have officially documented a need for an accommodation will have their request honored. Information regarding diagnostic criteria and policies for obtaining disability-based academic accommodations can be found at [www.uta.edu/disability](http://www.uta.edu/disability) or by calling the Office for Students with Disabilities at (817) 272-3364.

**Title IX:** The University of Texas at Arlington is committed to upholding U.S. Federal Law "Title IX" such that no member of the UT Arlington community shall, on the basis of sex, be excluded from participation in, be denied the benefits of, or be subjected to discrimination under any education program or activity. For more information, visit [www.uta.edu/titleIX](http://www.uta.edu/titleIX).

**Academic Integrity:** Students enrolled in this course are expected to adhere to the UT Arlington Honor Code:

*I pledge, on my honor, to uphold UT Arlington's tradition of academic integrity, a tradition that values hard work and honest effort in the pursuit of academic excellence. I promise that I will submit only work that I personally create or contribute to group collaborations, and I will appropriately reference any work from other sources. I will follow the highest standards of integrity and uphold the spirit of the Honor Code.*

UT Arlington faculty members may employ the Honor Code as they see fit in their courses, including (but not limited to) having students acknowledge the honor code as part of an examination or requiring students to incorporate the honor code into any work submitted. Per UT System *Regents' Rule* 50101, §2.2, suspected violations of university's standards for academic integrity (including the Honor Code) will be referred to the Office of Student Conduct. Violators will be disciplined in accordance with University policy, which may result in the student's suspension or expulsion from the University. *Papers involving plagiarism will receive an indelible grade of zero.*

**Electronic Communication:** UT Arlington has adopted MavMail as its official means to communicate with students about important deadlines and events, as well as to transact university-related business regarding financial aid, tuition, grades, graduation, etc. All students are assigned a MavMail account and are responsible for checking the inbox regularly. There is no additional charge to students for using this account, which remains active even after graduation. Information about activating and using MavMail is available at <http://www.uta.edu/oit/cs/email/mavmail.php>.

To obtain your NetID or for logon assistance, visit <https://webapps.uta.edu/oit/selfservice/>. If you are unable to resolve your issue from the Self-Service website, contact the Helpdesk at [helpdesk@uta.edu](mailto:helpdesk@uta.edu) or (817)272-2208.

**Student Feedback Survey:** At the end of each term, students enrolled in classes categorized as lecture, seminar, or laboratory will be asked to complete an online Student Feedback Survey (SFS) about the course and how it was taught. Instructions on how to access the SFS system will be sent directly to students through MavMail approximately 10 days before the end of the term. UT Arlington's effort to solicit, gather, tabulate, and publish student feedback data is required by state law; student participation in the SFS program is voluntary.

**Final Review Week:** A period of five class days prior to the first day of final examinations in the long sessions shall be designated as Final Review Week. The purpose of this week is to allow students sufficient time to prepare for final examinations. During this week, there shall be no scheduled activities such as required field trips or performances; and no instructor shall assign any themes, research problems or exercises of similar scope that have a completion date during or following this week *unless specified in the class syllabus*. During Final Review Week, an instructor shall not give any examinations constituting 10% or more of the final grade, except makeup tests and laboratory examinations. In addition, no instructor shall give any portion of the final examination during Final Review Week. During this week, classes are held as scheduled. In addition, instructors are not required to limit content to topics that have been previously covered; they may introduce new concepts as appropriate.

**Grade Grievances:** Any appeal of a grade in this course must follow the procedures and deadlines for grade-related grievances as published in the current graduate catalog.

**Emergency Exit Procedures:** Should we experience an emergency event that requires us to vacate the building, students should exit the room and move toward the nearest exit. When exiting the building during an emergency, one should never take an elevator but should use the stairwells. Faculty members and instructional staff will assist students in selecting the safest route for evacuation and make arrangements to assist handicapped individuals.

**Student Support Services:** UTA provides a variety of resources and programs designed to help students develop academic skills, deal with personal situations, and better understand concepts and information related to their courses. Resources include tutoring, major-based learning centers, developmental education, advising and mentoring, personal counseling, and federally funded programs. For individualized referrals, students may contact the Maverick Resource Hotline by calling 817-272-6107, sending a message to [resources@uta.edu](mailto:resources@uta.edu), or visiting [www.uta.edu/resources](http://www.uta.edu/resources).