MATH 5305: Assignment 1

1. Let A_1 and A_2 be two events related to an experiment. Given

$$P(A_1) = \frac{1}{2}, P(A_2) = \frac{1}{3}, P(A_1 \cap A_2) = \frac{1}{4}$$

Find the following probabilities

- (a) $P(A_1 \cup A_2)$ [**3 marks**]
- (b) $P(A_1^c \cup A_2^c)$ [**3 marks**]
- (c) $P(A_1^c \cap A_2^c)$ [**3 marks**]
- (d) $P(A_1^c \cup A_2)$ [**3 marks**]
- (c) $P(A_1^c \cap A_2)$ [**3 marks**]

2. Let $P(B) > 0$ so that $P(A|B)$ is defined.

- (a) If $A \subset B$, what is $P(A|B)$? [**3 marks**]
- (b) Show that if $P(A) = 0$, then $P(A|B) = 0$ [**3 marks**]
- (c) Show that if $P(A) = 1$, then $P(A|B) = 1$ [**3 marks**]

3. If A and B are independent events, show the following

- (a) A and B^c are independent [**5 marks**]
- (b) A^c and B^c are independent [**5 marks**]

4. A fair die is thrown 10 times. What is the probability of getting 6 in 4 throws? [**5 marks**]

5. What is the probability that the birthdays of 7 people will fall on 7 different days of the weeks, assuming equal probabilities for the seven days? [**5 marks**]

6. From a deck of 52 cards, 3 cards are drawn at random. Find the following probabilities

- (a) The cards are of the same denomination [**5 marks**]
- (b) 2 are of the same denomination and 1 different [**5 marks**]
- (c) All are of different denominations [**5 marks**]

7. A box contains 40 envelopes of which 25 are ordinary and 16 are unstamped. The number of unstamped ordinary envelopes is 10. What is the probability that an envelope chosen at random from the box is a stamped non-ordinary envelope? [**6 marks**]

8. Let A and B be two events with $P(A) = 0.4$, $P(B) = p$, and $P(A \cup B) = 0.7$.

(a) Find the value of p for which A and B are mutually exclusive [**5 marks**]

(b) Find the value of p for which A and B are independent [**5 marks**]

9. It has been found from past experience that of the articles produced by a factory, 20% come from Machine 1, 30% come from Machine 2, and 50% come from Machine 3. The percentages of satisfactory articles among those produced are 95% for Machine 1, 85% for Machine 2, and 90% for Machine 3.

(a) An article is chosen at random from a lot. What is the probability that it is satisfactory? [**5 marks**]

(b) Assuming that the article is satisfactory, what is the probability that it was produced by Machine 1 [**5 marks**]

10. 5 non-similar pairs of socks are in a closet. 4 socks are chosen at random. What is the probability that there will be among the 4 socks chosen

(a) No complete pair [**5 marks**]

(b) Exactly 1 complete pair [**5 marks**]

(c) Exactly 2 complete pairs [**5 marks**]

$$* * * * *$$

Answers to selected questions

4. 0.054

5. 0.006

6. (a) 0.002, (b) 0.169, (c) 0.828

7. 0.225

10. (a) 0.381, (b) 0.571, (c) 0.048