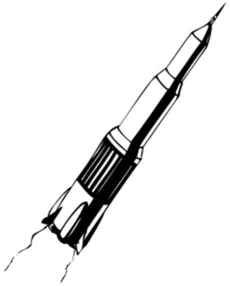
Spacecraft Propulsion, Launch and Orbit Design

Rocket Scientists Gift to Exoplanet Scientists



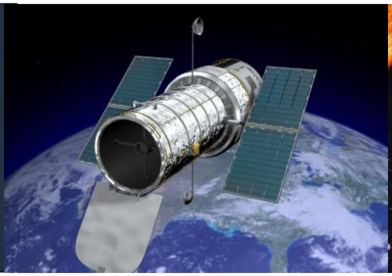
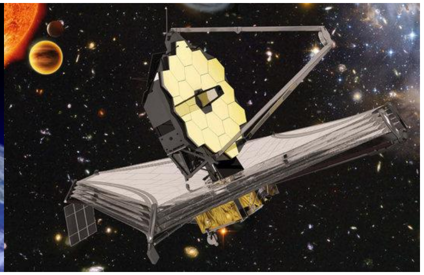
Physics Department Invited Lecture

Dora E. Musielak, Ph.D.

University of Texas at Arlington

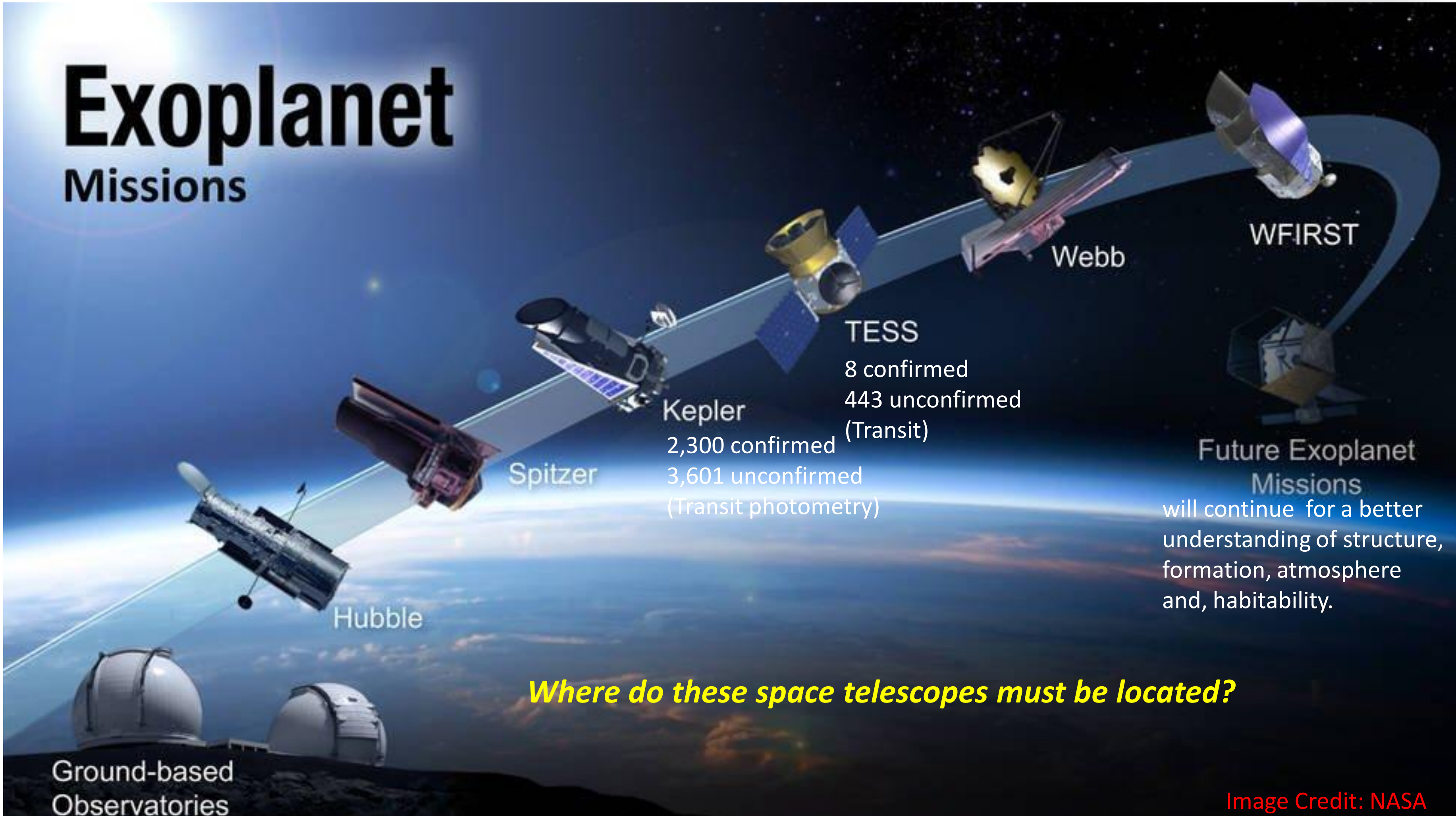
1 April 2019



4,023 confirmed exoplanets …

***most exoplanets have been discovered through space observatories*.**



TESS Launch and Orbit Deployment

On 18 April 2018, SpaceX Falcon 9 launched TESS spacecraft into a Suprasynchronous Transfer Orbit, 200 x 270,000 km at *i* = 28.5°. A 4 m/s burn for 54 sec was done to lift perigee to 600 km. TESS deployed into highly elliptical orbit ~48 minutes after liftoff.

LAUNCH AND TESS DEPLOYMENT https://www.youtube.com/watch?v=TfZ2y3-2tc0

Hour/Min/Sec	Events
00:00:00	Falcon 9 liftoff (Thrust: 7,607 kN)
00:01:16	Max q (peak mechanical stress on vehicle)
00:02:29	**1st stage MECO**
00:02:32	1st and 2nd stages separate
00:02:34	**2nd stage starts**
00:03:05	**Fairing deployment**
00:06:29	1st stage entry burn
00:08:17	**2nd stage SECO-1**
00:08:20	1st stage landing
00:40:50	**2nd stage restarts**
00:41:49	**2nd stage SECO-2**
00:48:42	**TESS deployment**

Wait, this is not the whole story!

What happened to TESS after rocket launch deployment?

Non-linear equations of motion ➔ challenging mathematically ➔ technologically difficult

Satellites in suprasynchronous orbits spiral outward.

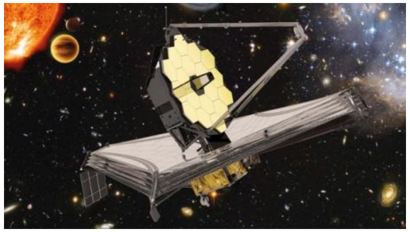
Outline

- **Exoplanet Discovery Missions**
- **Exoplanet Search Mission Orbit Requirements**
- **Optimizing Spacecraft Launch**
  - Space Launch Vehicle Constraints (telescope/spacecraft size, mass, vibration, thermal, …)
  - External Constraints (gravitational field, atmospheric drag, Earth oblatness, launch site, …)
- **Basic Concepts of Rocket Science**
  - Chemical Rocket Propulsion (Liquid Propellant Rocket Engine, Solid Propellant Rocket Motor, Thruster (monopropellant, bipropellant, cold gas, etc.)
  - Ideal Rocket Equation and Rocket Propulsion Performance Measures
  - Rocket Space Launch Capability (for Exoplanet Spacecraft)
- **Two-Body Problem ➔ Keplerian Motion**
  - Spacecraft Orbit Types (Closed, Hyperbolic, Escape, Resonant)
  - Trajectory Type and Class
  - Sphere of Influence
  - C_3 and Infinite Velocity (Hyperbolic Excess Velocity)
- **Restricted Three-Body Problem (R3BP)**
  - Euler-Lagrange Equilibrium Points (Sun-Earth and Earth-Moon Systems)
  - Lissajous Trajectories and Halo Orbits
  - Kosai-Lidov Mechanism
- **Examples of Spacecraft Orbits for Exoplanet Hunting Missions**
  - TESS
  - KEPLER
  - JWST
  - WFIRST
- **Spacecraft Orbit Design Approach, Optimization and Design Tools**

Exoplanet Discovery Missions

Transiting Exoplanet Survey Satellite (TESS) : MIT/NASA Exoplanet Explorer designed to perform first-ever spaceborne all-sky survey of exoplanets transiting bright stars. Monitoring over 200,000 stars for transits, and expect to find more than 1,600 planets smaller than Neptune. Launched 18 April 2018 on a 2 year survey mission.

Kepler : NASA Discovery mission launched on 6 March 2009, first space mission dedicated to search for Earth-sized and smaller planets in habitable zone of stars in neighborhood of our galaxy. Kepler spacecraft measured light variations from thousands of distant stars, looking for planetary transits. Kepler observed 530,506 stars and detected 2,662 planets. Intended for a 3.5 year long mission, Kepler remained operational over 9 years. Formally decommissioned 15 November 2018.



James Webb Space Telescope (JWSP) : Successor to Hubble Space Telescope, JWSP is intended for a broad range of investigations across astronomy and cosmology. Among its mission objectives, JWSP will study atmospheres of known exoplanets and find some Jupiter-sized exoplanets with direct imaging.

Wide Field Infrared Survey Telescope (WFIRST): Expected to launch in mid-2020s on 5-year mission, WFIRST will search for and study exoplanets while providing clues to detect dark matter.

Where do these space telescopes must be located?

Exoplanet Mission Orbit Requirements

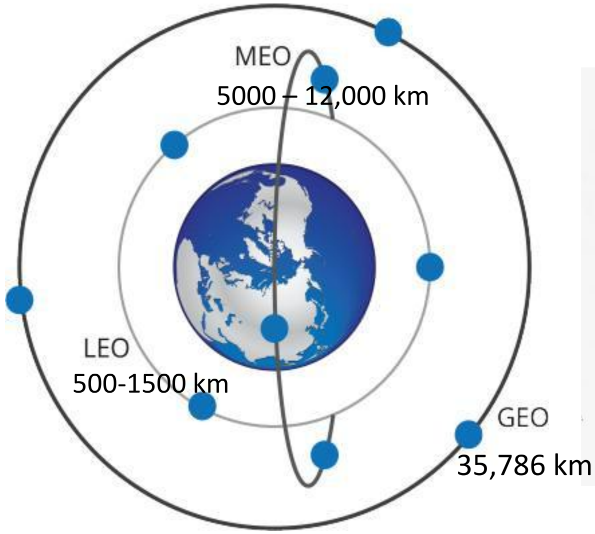
Ideal Orbit:

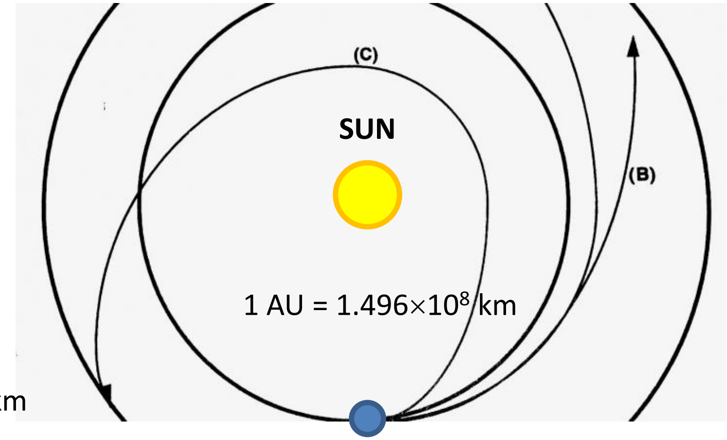
- **Allow for continuous observations lasting for *x* time (days, weeks, …)**
- **Be stable to perturbations over a multi-year mission duration.**
- **Offer a low-radiation environment, to avoid high trapped-particle fluxes and resulting degradation of detectors and light electronics.**
- **Offer a stable thermal environment and minimal attitude disturbance torques, to provide a stable platform for precise photometry.**
- **Orbit achievable with a moderate Δv , avoiding need for a costly secondary propulsion unit.**
- **To facilitate data transfer, spacecraft to be close to Earth during at least a portion of its orbit.**

COROT: geocentric polar orbit
(r_p = 607.8 km, r_a = 898.1 km)

HUBBLE: LEO orbit
Altitude: 590 km
***i* = 28.5°**
Period: 96-97 min

WFIRST: heliocentric halo orbit about ESL2
(r_p = 188,420 km, r_a = 806,756 km)





High Earth Orbits (HEO)

High Earth orbit (HEO): geocentric orbit with an altitude entirely above that of a geosynchronous orbit.

Advantages:

- **Away from Earth's radiation Belts,**
- **Small gravity gradient effects,**
- **No atmospheric torques,**
- **Perigee altitudes remaining above geosynchronous altitude ($r_p > 6\ R_E$),**
- **Excellent coverage by a single ground station,**
- **Modest launch vehicle and spacecraft propulsion requirements (using lunar gravity assist),**
- **More launch opportunities per month than a lunar swingby L2 point mission (greater flexibility in choice of Moon location at lunar swingby.**

Geocentric Orbit radius: $r = R_E + h$

h = altitude over surface of planet

Exoplanet Mission Orbit Requirements

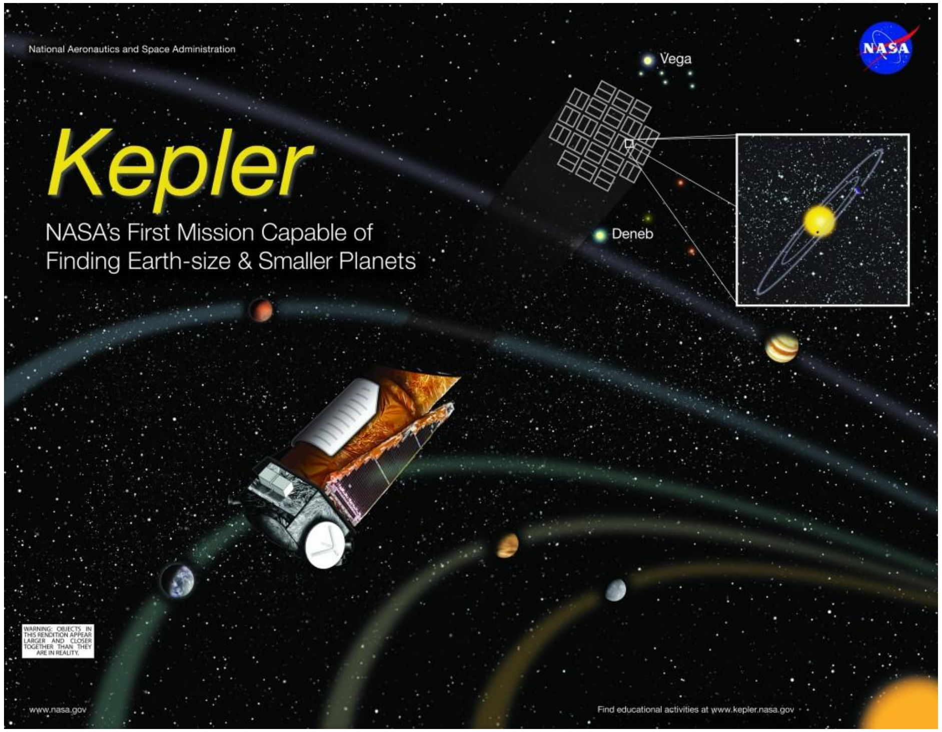
Ideal Orbit Requirements are specific to mission:

- **Kepler telescope's mission to find Earth-size planets in HZ—potential abodes for life—required continuous viewing of star fields* near galactic plane.**

- **It required photometer field-of-view to be out of ecliptic plane so as not to be blocked periodically by Sun or Moon.**

- **Gravity gradients, magnetic moments and atmospheric drag cause spacecraft torques, which lead to unstable pointing attitude.**

- **Heliocentric Earth-trailing orbit selected for Kepler spacecraft.**



KEPLER: heliocentric orbit $r_p = 0.97671 \text{ AU}$ $r_a = 1.0499 \text{ AU}$

r_p = perihelion, r_a = aphelion
AU = astronomical unit; 1 AU = 149597870700 m = 1.496 x 10^8 km
1 AU = 4.8481×10^{-6} pc; 1.5813×10^{-5} ly

* To measure dip in brightness as a planet passes in front of its host star

Optimizing Spacecraft Launch

Exoplanet spacecraft require to be launched by rocket propulsion to overcome Earth's attraction and gain huge speed needed to orbit central body (Earth or Sun, or equilibrium point in R3BP).

Spacecraft Launch mass: 1039 kg

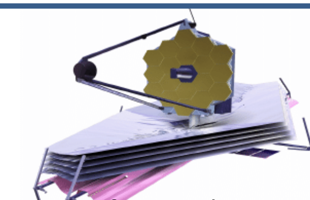
Launch: 7 March 2009
Mission end: 30 Oct 2018

Delta II

Spacecraft Launch Mass: 362 kg

Launch: 18 April 2018

Falcon 5



Spacecraft Launch Mass: 6200 kg

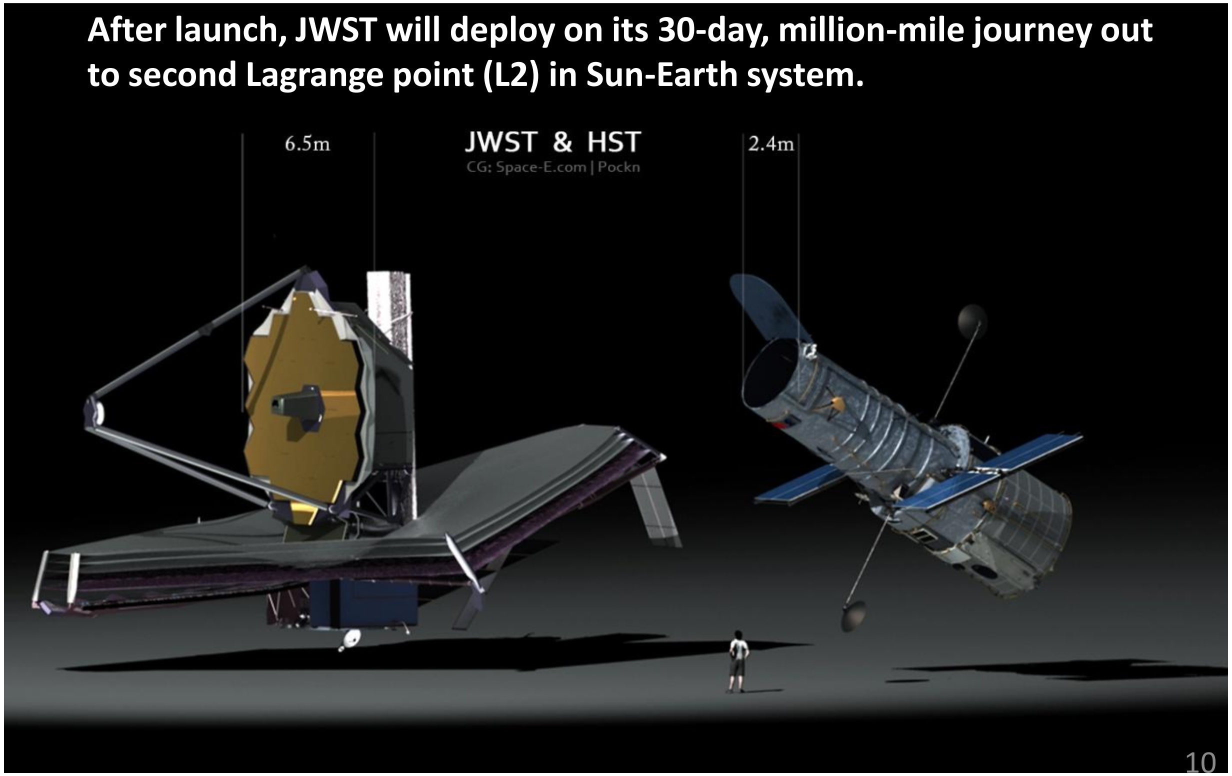
Launch: March 2021

Ariane 5

Payload Imposes Constraints on Rockets

Let's get a perspective on spacecraft to be launched …

Lifting a spacecraft to LEO altitud (e.g. 300 km) requires energy ➔ mighty rocket launch vehicle.
Staying in orbit requires even more energy ➔ spacecraft have built-in thrusters for orbit control.



Payload Constraints to Launch

Depending on payload, size/volume of launch vehicle fairing will vary, and also number of strap-on rocket motors

Ariane 5 Payload Fairing
diameter = 4.57 m
length = 16.19 m

Payload Fairing: a nose cone used to protect a spacecraft (launch vehicle payload) against impact of dynamic pressure and aerodynamic heating during launch through an atmosphere.

Image: https://jwst.nasa.gov/launch.html

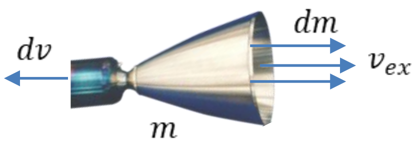
Rocket Science Supports Exoplanet Science

- ➢ To launch a spacecraft from Earth into orbit requires increasing its specific mechanical energy. This energy comes from powerful rockets propelling launch vehicle.
- ➢ To keep a spacecraft in orbit and maneuver in space requires efficient propulsion system.
- ➢ Spacecraft maneuver design is reflected in propellant mass required to accomplish it.

- Launch Analysis ➔Spacecraft are launched into orbit by chemical rocket propulsion ➔ launch vehicles are configured to meet requirements for each mission (payload size, orbit elements, etc.)
- Mission Analysis ➔ Mid-course Correction (MCC) maneuvers during launch and early orbit phase and transfer require efficient propulsion.
- Reaction Control System (RCS) ➔ uses thrusters to provide attitude control, and translation.
    - attitude control during entry/re-entry;
    - stationkeeping in orbit;
    - close maneuvering during docking procedures;
    - control of orientation;
    - a backup means of deorbiting;
    - ullage motors to prime fuel system for a main engine burn.
- Rocket Equation ➔ propellant mass necessary to produce velocity change or Δv.
- Delta-v and stationkeeping analysis ➔ determine how much propellant to carry to complete spacecraft mission.

Rocket Thrust and Launch Analysis

$$m\frac{dv}{dt} + v_{ex}\frac{dm}{dt} = 0$$



$$\Delta v = -\int_{t_0}^{t_1} v_{ex}\frac{\dot{m}}{m}dt \Rightarrow \boxed{\Delta v = v_{ex}\ln(m_i/m_f)}$$

Ideal Rocket Equation

Spacecraft m_i, m_f are initial and final masses before and after propellant is burned.

m = mass of spacecraft at any instant

Thrust force on spacecraft = change in momentum of exhaust gas, which is accelerated from rest to velocity v_{ex}

Thrust: $$F = v_{ex}\left(\frac{dm}{dt}\right) = \dot{m}v_{ex}$$

Exhaust gas velocity is limited for chemical rockets :

$$v_{ex} \propto \sqrt{T} \approx 4600 \text{ m/s (LH2 + LOX)}$$

Specific Impulse

$$I_{sp} = F/g_0\dot{m} = v_{ex}/g_0$$

$$\Delta v = g_0 I_{sp}\ln(m_i/m_f)$$

Require huge thrust for liftoff and reach orbit velocity!

I_{sp} = thrust produced by a unit propellant weight flow rate

Propellant	I_{sp}, s
Cold gas	50
Monopropellant hydrazine	230
Solid propellant	290
Nitrogen tetroxide/MMH	310
Liquid oxygen/liquid hydrogen	460

Launch Analysis: To illustrate it, consider just a small portion (vertical flight).

1-D equation of motion:

$$m\frac{dv}{dt} = v_{ex}\frac{dm}{dt} - \left(C_D A\frac{\rho v^2}{2}\right) - mg \Rightarrow \boxed{(m_0 - \dot{m}t)\frac{dv}{dt} = v_{ex}\frac{dm}{dt} - \left(C_D A\frac{\rho v^2}{2}\right) - (m_0 - \dot{m}t)g}$$

$$m = (m_0 - \dot{m}t) \qquad g = g_0\left(\frac{r_0}{r}\right)^2 = g_0\left(\frac{r_0}{r_0 + h}\right)^2 \qquad g_0 = 9.8 \text{ m/s}^2$$

Delta-v for Propulsive Maneuvers

- **Delta-v budget analysis determines propulsion demand for given mission.**
- **We use delta-v budget as indicator of how much propellant will be required.**

$$\Delta v = g_0 I_{sp} \ln(m_i/m_f)$$

If thruster mass fraction is 20% and has constant v_{ex}=2100 m/s (typical for hydrazine thruster), delta-v capacity of reaction control system (RCS) is $\Delta v = 2100 \ln(1/0.8) = 460 \text{ m/s}$

Since Δv is known, rearrange rocket equation to obtain mass of propellant m_p consumed to produce given velocity change:

$$m_p = m_i \left[1 - \exp\left(-\frac{\Delta v}{g_0 I_{sp}}\right)\right] = m_f \left[\exp\left(\frac{\Delta v}{g_0 I_{sp}}\right) - 1\right]$$

Maneuver Type	Orbit altitude (km)	Δv per year (m/s)
Maintain position		50 – 55
Maintain orbit	400 – 500	< 25
Maintain orbit	500 – 600	< 5
Maintain orbit	> 600	
Maintain orientation		2 – 6
Change from circular to elliptical orbit (plane)	300 to 300 x 3000	624 (one time)

Propellant usage is exponential function of delta-v in accordance with rocket equation!

Rocket Thrust for Spacecraft Launch

SpaceX Falcon 9 Launch mass: 549,054 kg (1,210,457 lbm)

981 kN (Vac)
(220,500 lbf)

7,686 kN (SL)
(1,710,000 lbf)

Payload

LOX Tank

RP-1 Fuel Tank

LOX Tank

RP-1 Fuel Tank

Images: SpaceX

Require huge thrust for liftoff!

$$F > g_0 m_{vehicle}$$

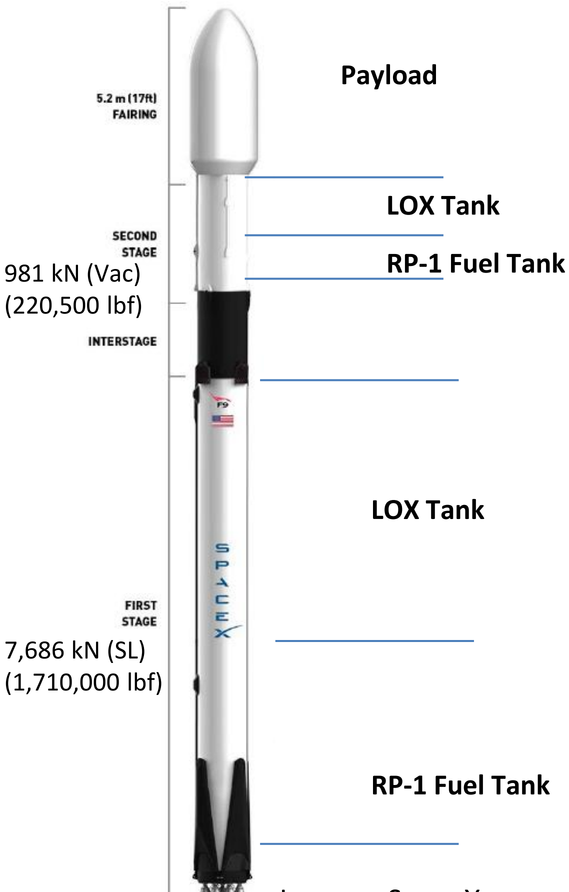
And to take vehicle and its payload from zero to orbital velocity! $\Delta v = g_0 I_{sp} \ln(m_i/m_f)$

Second stage: Merlin engine. It delivers payload to right orbit
Thrust: 934 kN (210,000 lbf)
I_{sp}: 348 s
Burn time: 397 s

First Stage: 9 Merlin engines and Al-Li alloy tanks with LOX and rocket-grade kerosene (RP-1) propellant.
Thrust (SL): 7,607kN (1,710,000 lbf)
Thrust (vac): 8,227kN (849,500 lbf)
Burn Time: 162 s (time interval from ignition to cutoff, when rocket engine produces meaningful thrust).

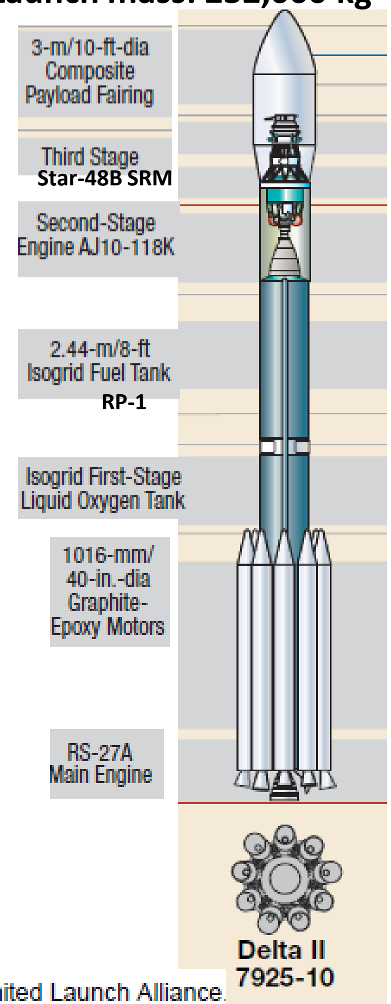
Rocket Space Launch Capability

Falcon 9 Launch mass: 549,054 kg (1,210,457 lbm)



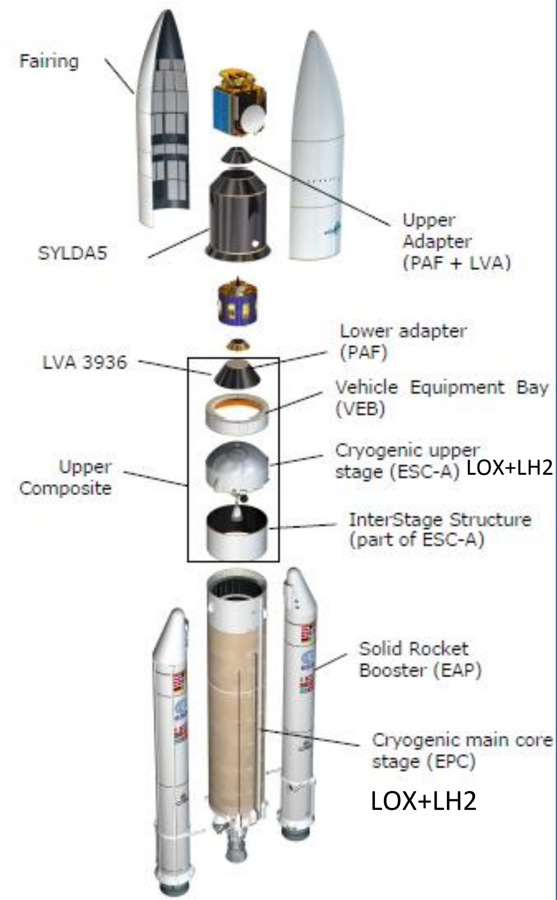
Images: SpaceX

ULA Delta II 7925-10L Launch mass: 232,000 kg



United Launch Alliance

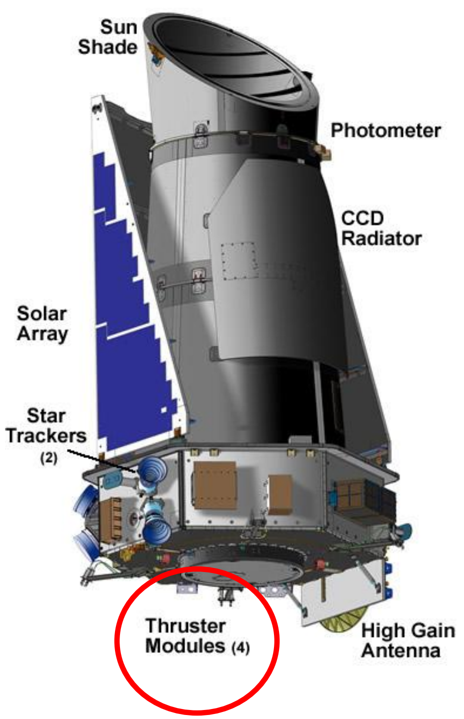
Arianspace Ariane 5 ES Launch mass: 777,000 kg (750 t)



Arianspace Ariane 5 Users Manual

Mission Propulsion Requirement

Propulsion provides the means to correct spacecraft's orbit and to control attitude in certain operating modes.
Spacecraft thrusters help correct drift and perform orbital maneuvers, including pointing to new fields of view and orienting its transmitters to Earth to downlink science data and receive commands.



Spacecraft must carry enough propellant to last entire mission.

Kepler launched on 7 March 2009 with 12 kg (~3 gal) of hydrazine in its fuel tank.

On 30 Oct. 2018, NASA confirmed,
Kepler ran out of gas!

Primary mission was planned to be 3.5 years. To ensure that Kepler would operate for up to 6 years, engineers estimated to carry 7 to 8 kg of fuel, and designed an oversized fuel tank, which would be partially filled. As spacecraft was prepared for launch, it weighed less than max weight rocket launcher could lift. Engineers decided to fully fill Kepler's fuel tank. This extended fuel estimated life to almost 10 years.

1 N monopropellant hydrazine thruster in Corot spacecraft

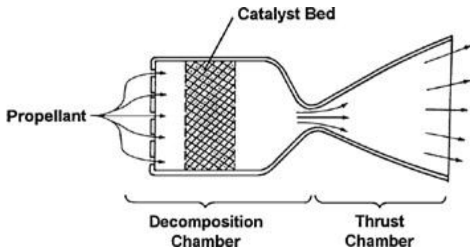
Spacecraft Thrusters

Thruster: a low thrust propulsive device used by spacecraft for station keeping, attitude control, in the reaction control system, or long-duration, low-thrust acceleration.

Spacecraft require monopropellant thrusters for attitude control, and bipropellant or bimodal Secondary Combustion Augmented Thrusters (SCATs) for velocity control.

Typical thruster is a monopropellant system with anhydrous hydrazine propellant and GN2 pressurant.

Monopropellant Thruster
Propellant: Hydrazine or Monomethyl Hydrazine (MMH)
Thrust: 1 – 4000 N (variable thrust)
Typical Application: Spacecraft reaction control (RCS)



$$F = \dot{m} v_{ex}$$

Monopropellant Thruster. Propellant passes through catalyst bed. Decomposition yields gaseous reaction products, which are expelled at high pressure through a CD nozzle to create thrust.

Aerojet Rocketdyne MR-104
Hydrazine (N_2H_4)
Thrust: 440 N Class
Mass: 1.86 kg

Bipropellant Thruster	Application	Propellant	Thrust (kN)	Isp (s)
Aerojet OMS-E	OMS	NTO/MMH	26.689	313 (v)
KM R-40	RCS	NTO/MMH	3.87 (range 3.114–5.338 kN)	280 (v)
KM R-1E	Attitude control and orbit adjust	NTO/MMH	110 N (range 67-155.7 N)	280 (v)

JWST Propulsion Requirements

Propulsion for James Webb Space Telescope (JWST) comprised of two types of thrusters:

(1) Secondary Combustion Augmented Thrusters (SCATs), main thrusters for Mid-Course Correction (MCC) maneuvers. One pair of SCATs is also planned for station keeping throughout mission.

(2) Dual Thruster Modules (DTMs) each comprising of a primary and redundant Monopropellant Rocket Engine, 1 lbf, (MRE-1) thruster.

Bi-Modal Thruster or Secondary Combustion Augmented Thrusters SCAT is a bipropellant-type thruster. Thruster can be used in a bipropellant mode when spacecraft maneuver requires higher thrust, and also in a monopropellant mode for lower thrust maneuvers.

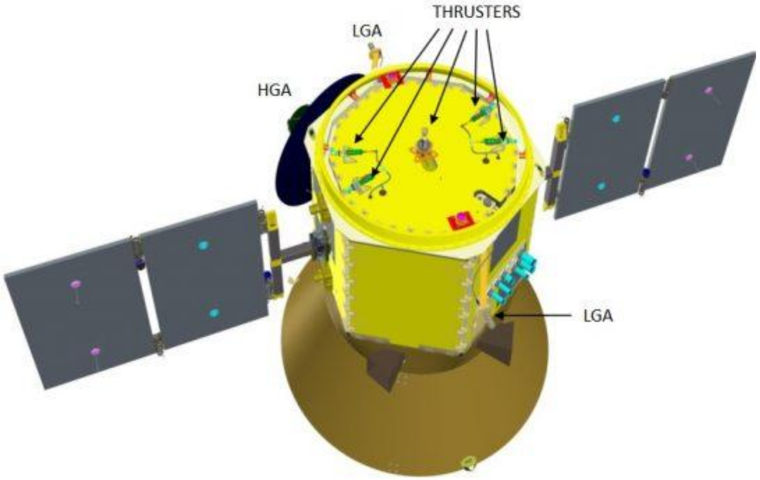
For example, during initial high-impulse orbit-raising maneuvers, system operates in a bipropellant fashion, providing high thrust at high efficiency; when spacecraft arrives to its orbit, it closes off either fuel or oxidizer, and conducts remainder of its mission in a simple, predictable monopropellant fashion.

More on JWST propulsion design:

https://jwst-docs.stsci.edu/display/JTI/JWST+Propulsion

Bipropellant Thruster
(Northrop Grumman)

TESS Orbit and Attitude Control



Four attitude control thrusters (Thrust=5 N each)
One orbital maneuvering thruster (Thrust = 22 N)

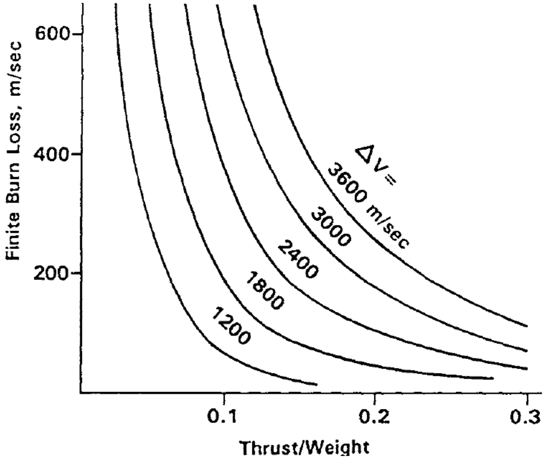
TESS includes a five Hydrazine Monopropellant Propulsion System for orbit and attitude control. Propellant fed from central tank containing 45 kg of hydrazine.

Orbit and Attitude Thrusters yield a total delta-v budget of 268 m/s.

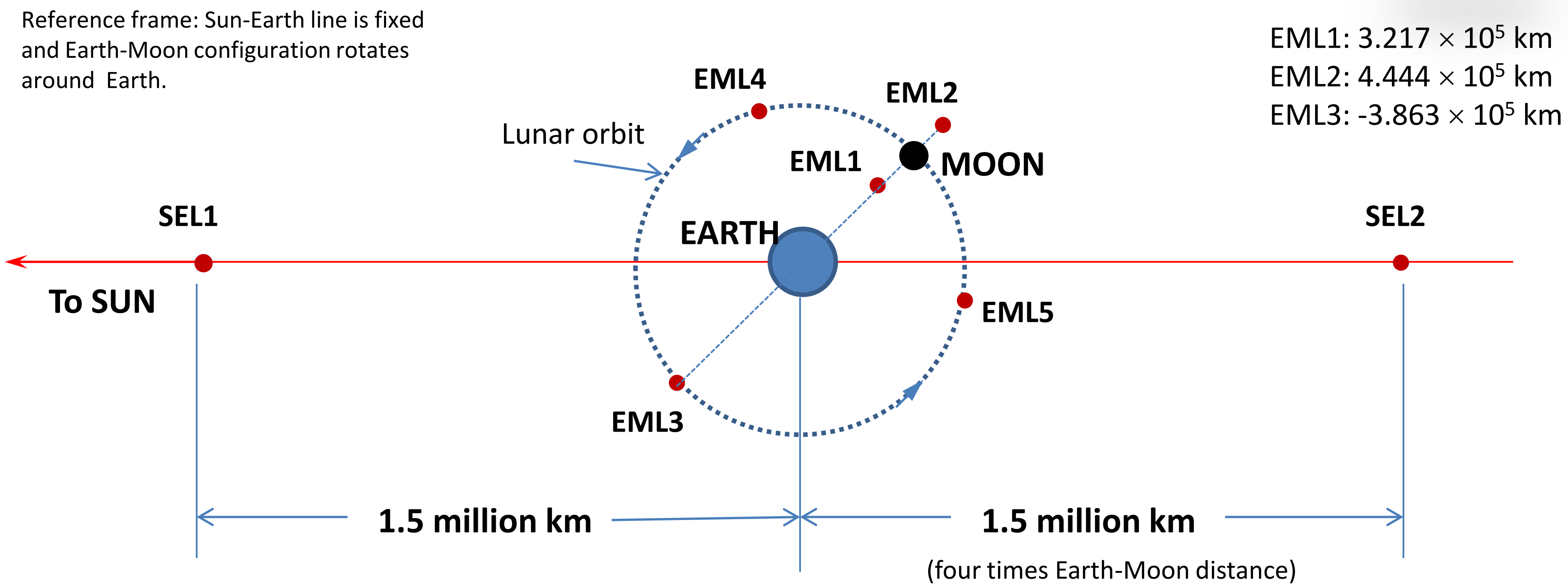
Finite Burn Losses

Analysis assumes that velocity is changed at a point on trajectory, i.e., that a velocity change is instantaneous. If assumption is not valid, serious energy losses can occur, known as *finite burn losses.*
It requires a numerical integration to evaluate. Low thrust-to-weight ratios (F/W) cause finite burn loss and should be avoided.

We analyze extent of finite burn losses when thruster has F/W < 0.5.



Spacecraft Motion in Sun-Earth-Moon System



Sun, Earth, and Moon can be used to form trajectories that have considerable practical value for exploration of space, including human space travel and exoplanet hunting spacecraft.

Near Earth, spacecraft orbits may be analyzed as 2-Body Problems or Restricted 3-Body Problems. In Sun-Earth-System we can conceive a large variety of unique orbits based on Restricted 4-Body Problem.

Concepts of Orbit Analysis

Two-Body Problem ➔ Keplerian Motion

Orbit Types (Closed, Hyperbolic, Resonant, Escape)
Trajectory Type and Class
Sphere of Influence
Patched Conic Approximation
C_3 and Hyperbolic Excess Velocity

Restricted Three-Body Problem (R3BP)

Euler-Lagrange Equilibrium Points (Sun-Earth and Earth-Moon Systems)
Lissajous Trajectories and Halo Orbits
Kosai-Lidov Mechanism

Spacecraft orbital maneuvering is based on fundamental principle that an orbit is uniquely determined by position and velocity at any point.

Changing velocity vector at any point instantly transforms trajectory to correspond to new velocity vector.

2-Body Problem (Keplerian Motion)

Equation of motion

$$\ddot{\mathbf{r}} = -\frac{\mu}{r^3}\mathbf{r}$$

Orbit equation defines path of *m* around *M*

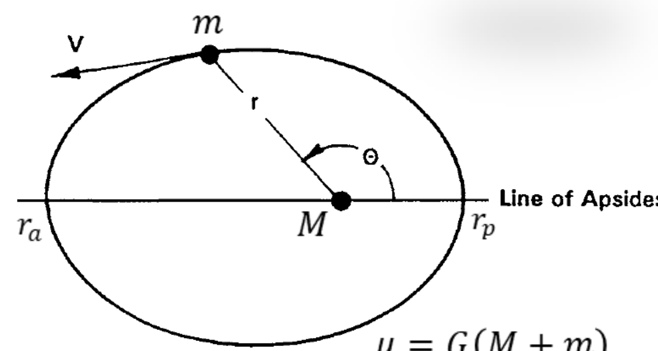
$$r = \frac{h^2}{\mu}\frac{1}{1+\cos\theta}$$

Position of *m* as a function of time (time from periapse passage)

$$\frac{\mu^2}{h^3}t = \int_0^{\theta}\frac{d\vartheta}{(1+e\cos\vartheta)^2}$$

Mean motion (body avg. angular velocity *ω* in elliptical orbit)

$$n \equiv \frac{2\pi}{T}$$



$$\mu = G(M+m)$$

when $m \ll M$ $\quad \mu = GM$

1) Kepler's laws of planetary motion confirmed and generalized to allow orbits of any conic section shape.
2) Sum of potential energy and kinetic energy of orbiting body, per unit mass, is a constant at all points in its orbit:

$$\boxed{\varepsilon = \frac{v^2}{2} - \frac{\mu}{r}}$$

Energy integral

Energy integral most useful relation from 2BP solution. It yields general relation for velocity of an orbiting body:

a = orbit semimajor axis
μ = gravitational parameter = *GM*

For a spacecraft orbiting Earth

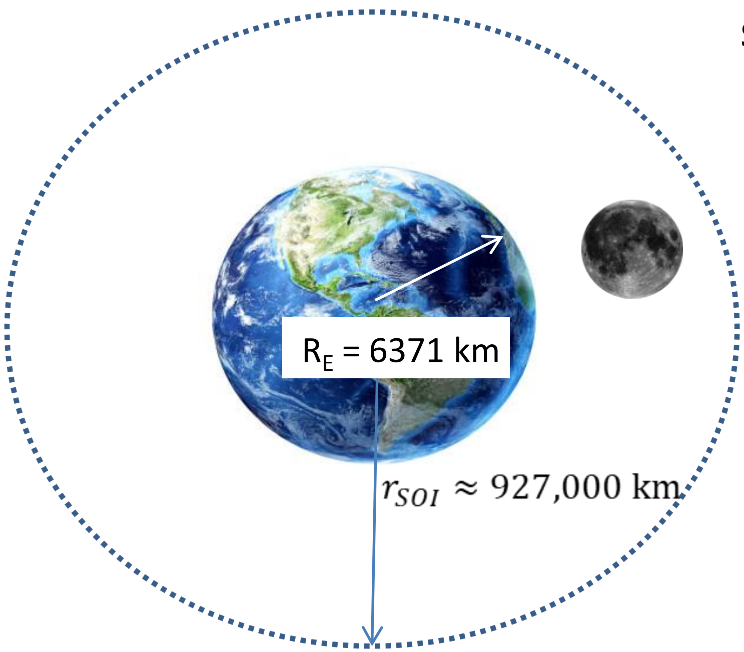
$$\mu = GM_E = 398{,}600.4\ \mathrm{km^3 s^{-2}}$$

$$\boxed{v = \sqrt{(2\mu/r) - (\mu/a)}}$$

Spacecraft velocity at any point on any orbit

Sphere of Influence (SOI)

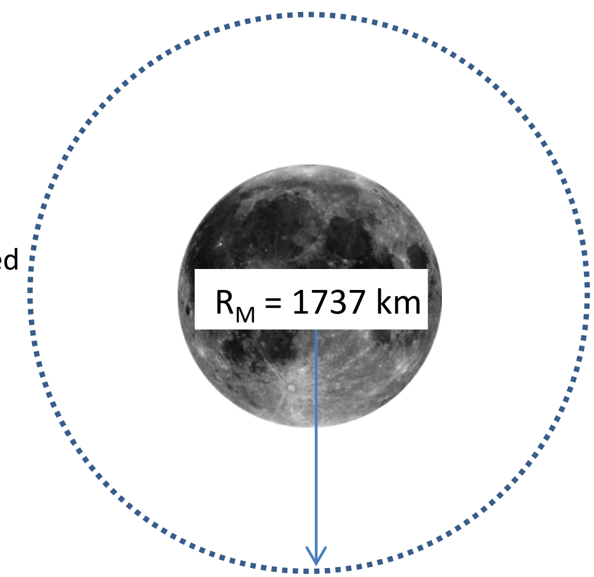
Near a given planet influence of its own gravity exceeds that of Sun. At Earth's surface gravitational force is over 1600 times greater than Sun's.
Within a planet's sphere of influence (SOI), spacecraft motion is determined by its equations of motion relative to planet (2-Body Problem)



Sphere of influence radius

$$r_{SOI} \approx r\left(\frac{m_p}{m_S}\right)^{2/5}$$

r = planet mean orbital radius (center-to-center distance between compared bodies, e.g., planet vs Sun, moon vs planet)



Earth's Radius of influence

$r_{SOI} \approx 145\ R_E$

For gravity boost spacecraft flyby 8047 km from Moon

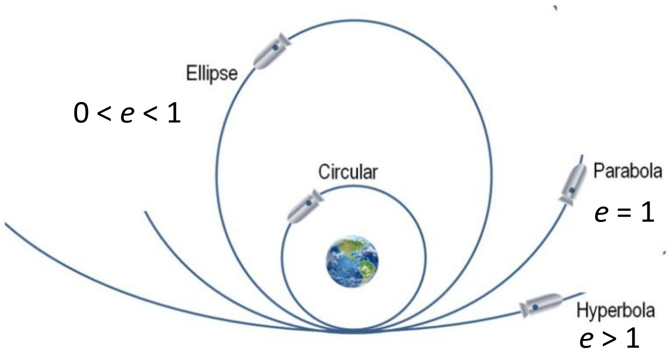
Moon's Radius of influence

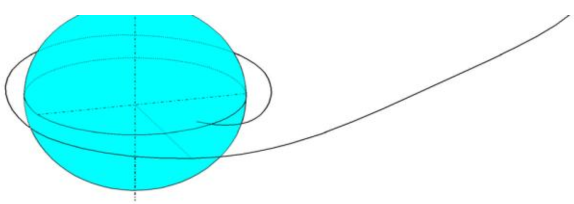
$r_{SOI} \approx 66{,}200$ km

Spacecraft Orbits

- Spaceflight is when a spacecraft is placed on a trajectory where it could remain in space for at least one orbit. For geocentric orbit, spacecraft's trajectory has altitude at perigee above 100 km (62 mi).

- To remain in orbit at this altitude requires an orbital speed of ~7.8 km/s. Orbital speed is slower for higher orbits, but attaining them requires greater delta-v.





Escape Orbit

specific energy $$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r}$$

numerical value of ε (+ or −) identifies spacecraft orbit:

+ ε ➔ spacecraft moves along a hyperbolic orbit.
ε = 0 ➔ path is a parabola.
– ε ➔ orbital path is either a circle or an ellipse.

Spacecraft Maneuvering Delta-v

Orbital maneuvering is based on fundamental principle that an orbit is uniquely determined by position and velocity at any point. Conversely, changing velocity vector at any point instantly transforms trajectory to correspond to new velocity vector. For example, to change from a circular orbit to an elliptical orbit, spacecraft velocity must be increased to that of an elliptic orbit.

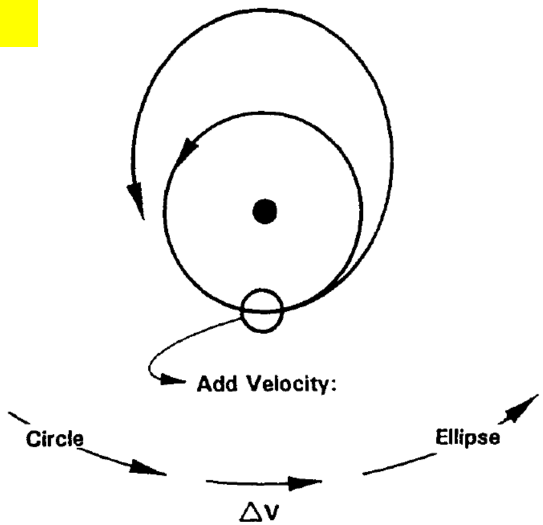
Spacecraft velocity should be increased at point of desired periapsis placement.

***Example*: A spacecraft with initial circular LEO at 300-km altitude must transfer to elliptical orbit with a 300-km altitude at perigee and a 3000-km altitude at apogee. Determine maneuver velocity increase (delta v)**

initial circular orbit $$V = \sqrt{\mu/r} = \sqrt{\frac{398{,}600.4}{(300 + 6378.14)}} = 7.726 \text{ km/s}$$

velocity at perigee $$V = \sqrt{(2\mu/r) - (\mu/a)} = 8.350 \text{ km/s}$$

Maneuver velocity increase: $$\Delta V = 8.350 - 7.726 = 0.624 \text{ km/s}$$



$$V(\text{circle}) + \Delta V = V(\text{ellipse})$$

Velocity changes made at periapsis change apoapsis radius but not periapsis radius, and vice versa. Plane of orbit in inertial space does not change as velocity along orbit is changed.

Minimum Energy Trajectory to Moon

What effect injection speed has on attaining lunar orbit? Assume that orbit injection into lunar trajectory occurs at perigee where $\phi o = 0°$.

Limiting case: injection speed (V_I) is infinite, path is a straight line with a time-of-flight of zero.

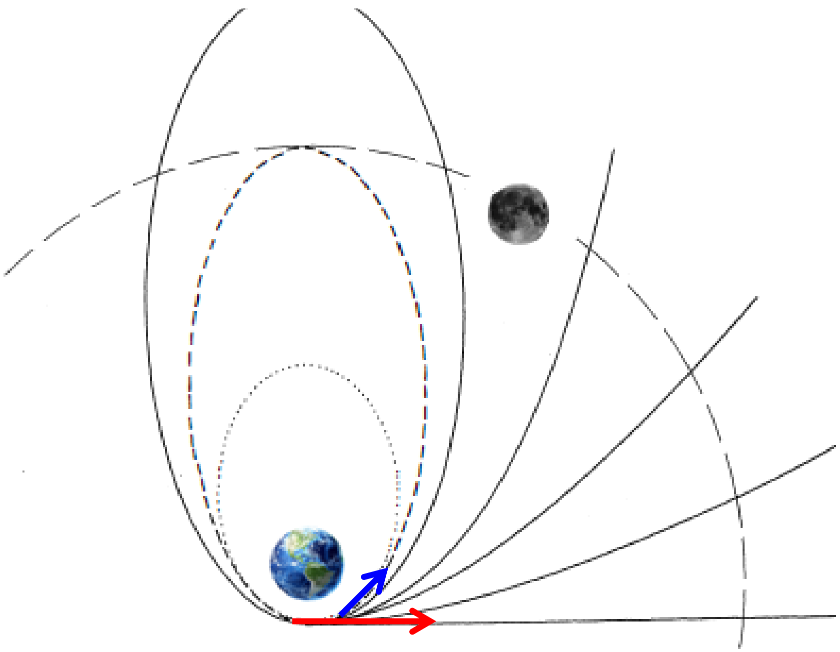
As we lower V_I orbit changes from hyperbolic, to parabolic, to elliptical in shape; time-of-flight increases.

If we keep reducing V_I, spacecraft will arrive at (dashed) orbit whose apogee just barely reaches to distance of Moon. This is minimum injection speed.

For injection at 320 km (0.05 R_E) altitude, this minimum injection speed is 10.82 km/sec.

If spacecraft moves at less than this speed it will fail to reach moon's orbit (dotted path)

Minimum energy trajectory is an ellipse just tangent to lunar orbit



To reach lunar orbit from 320 km altitude parking orbit, spacecraft requires injection speed V_I = 10.82 km/s.

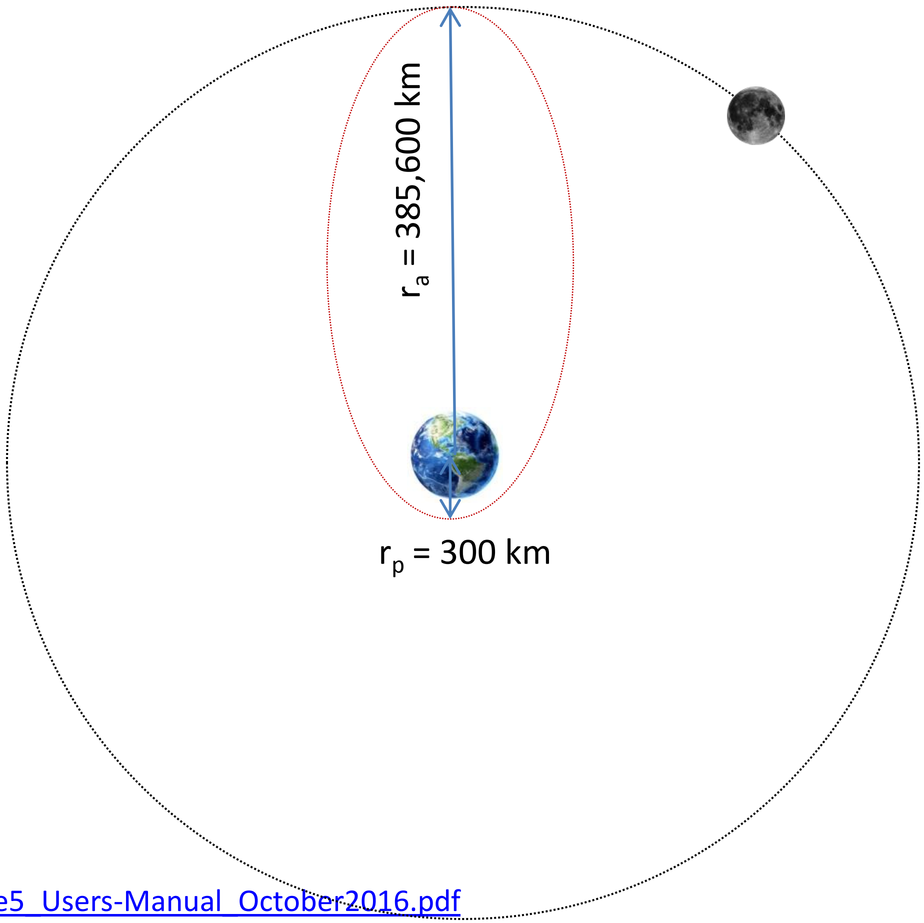
Launch/Injection (an Example)

Ariane 5 performance estimates with A5ECA for different elliptical missions (See User's Manual*):

Injection towards the Moon:
- apogee altitude 385,600 km
- perigee altitude 300 km
- inclination 12 deg
- performance 7 t

Earth escape missions (4550 kg)
Earth escape orbit:
- infinite velocity V_∞ = 3475 m/s
- declination δ = - 3.8°

Injection towards SEL2 Lagrangian point:
- apogee altitude 1,300,000 km
- perigee altitude 320 km
- inclination 14 deg
- argument of perigee 208 deg
- performance 6.6 t



* http://www.arianespace.com/wp-content/uploads/2011/07/Ariane5_Users-Manual_October2016.pdf

Departure Geometry/Velocity Vector

Launch Characteristic Energy C3

a measure of excess specific energy over that required to just barely escape from a massive body.

$$C3 = V_\infty^2$$

$$V_\infty = \sqrt{V_I^2 - \frac{2\mu}{r_I}}$$

V_∞ represents spacecraft velocity at a great distance from planet (where gravitational attraction is negligible).

V_∞ is attained when spacecraft has climbed away from departure planet, following injection at velocity V_I

$\mu = 398{,}600\ \mathrm{km^3/s^2}$

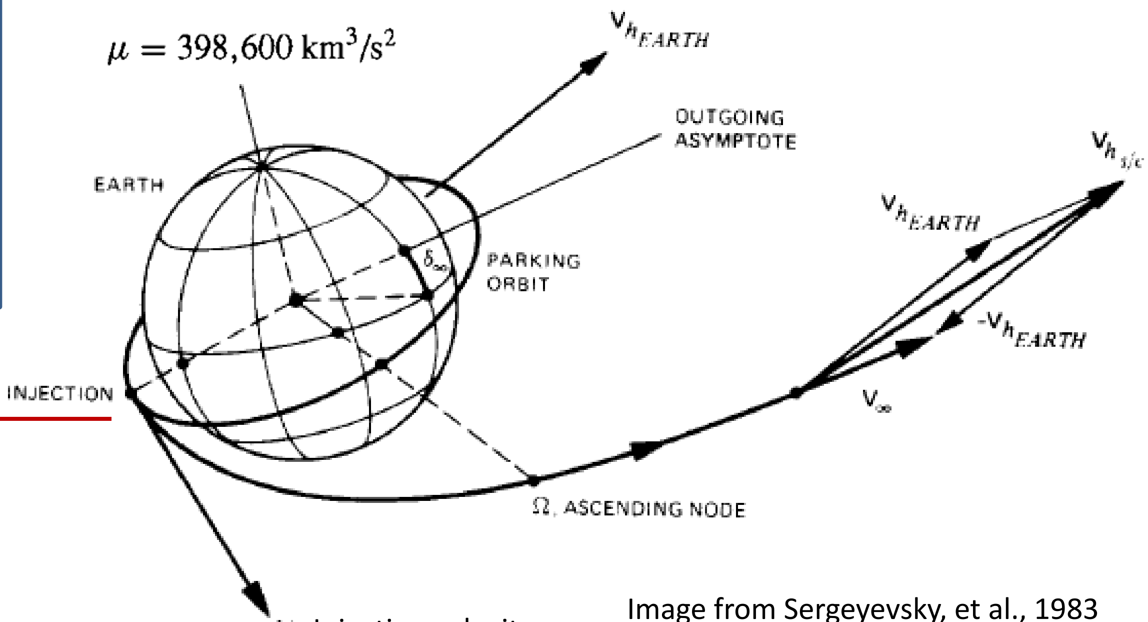


Image from Sergeyevsky, et al., 1983

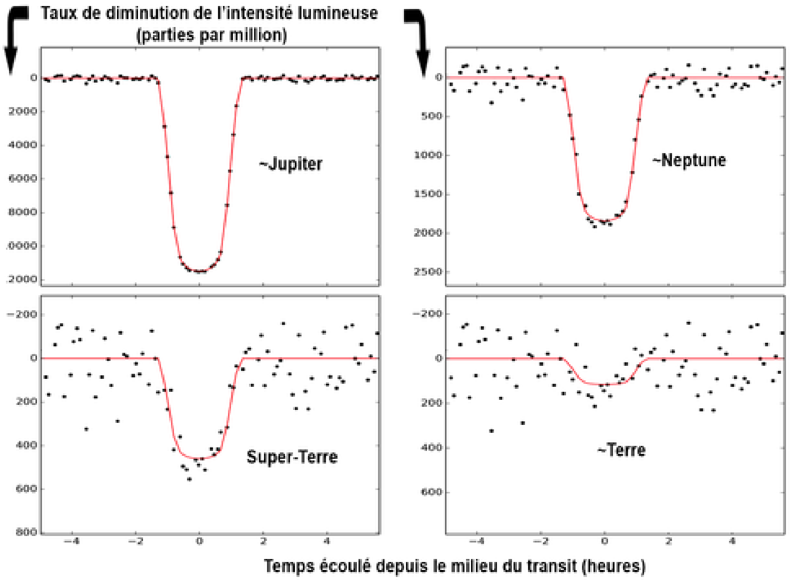
Ex. If injection is at $V_I = 11.05\ \mathrm{km/s}$ $r_I = 6554.3\ \mathrm{km}$

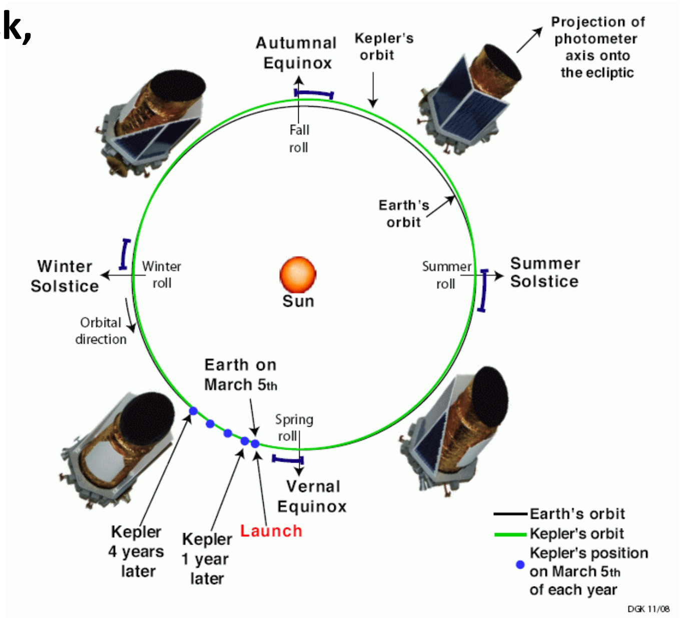
$$V_\infty = \sqrt{122.1025 - \frac{2(398{,}600)}{6554.3}} \qquad V_\infty = 0.7088\ \mathrm{km/s}$$

Kepler Exoplanet Hunter

NASA's first planet-hunting spacecraft launched 6 March 2009 on 3.5-year mission to seek signs of other Earth-like planets. Hugely successful, Kepler operated for nine years and discovered hundreds of exoplanets!

Kepler used transit photometric method to determine exoplanet's radius. If a planet crosses (transits) in front of its parent star's disk, then observed visual brightness of star drops by a small amount, depending on relative sizes of star and planet.





https://www.youtube.com/watch?v=54fnbJ1hZik

Kepler's orbit. Telescope's solar array was adjusted at solstices and equinoxes.

Injection Escape Trajectory

Primary problem in departure trajectory design is to match mission-required outgoing V_∞ vector to specified launch site location on rotating Earth.

Outgoing vector specified by its energy magnitude $C3 = |V_\infty|^2$, twice kinetic energy (per kg of injected mass) which must be matched by launch vehicle capability, and V_∞ direction with respect to inertial Earth Mean Equator: declination (i.e., latitude) of outgoing asymptote δ_∞ (DLA), and its right ascension (i.e., equatorial east longitude from vernal equinox) α_∞ (RLA).

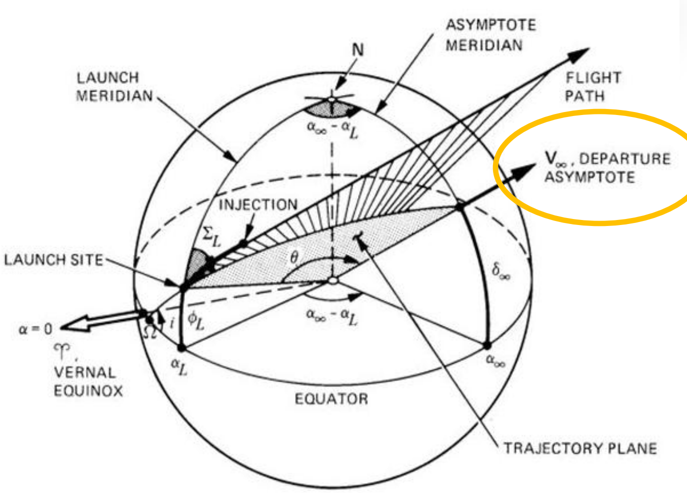


Image from Sergeyevsky, et al., 1983

Launched by Delta II, **Kepler spacecraft first launched into circular LEO 185.2 km x 185.2 km, *i* = 28.5° (parking orbit).** Vehicle+spacecraft coasted for ~ 43 min before reaching proper position to begin second, Earth-departure sequence.

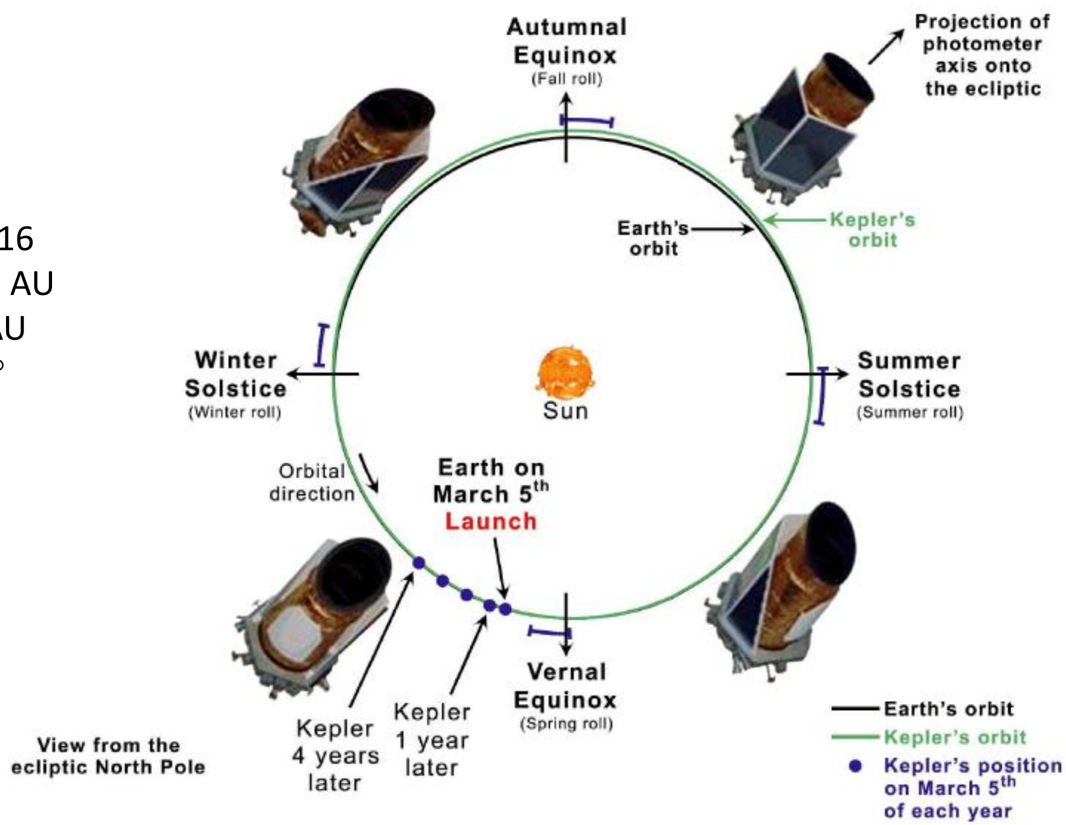
Third stage SRM ignited, taking ~ 90-s to burn 2,010 kg (4,431 lbm) of solid propellant, avg. F = 66,000 N. **Five minutes after third stage burn out, Kepler separated and escaped to its heliocentric Earth trailing orbit**.

For Kepler, at target intersect point
r_p = 183.3 +6371 = 6554.3 km
C3 = 0.60 km^2/s^2 $\quad V_\infty = 0.7088$ km/s
DLA = 23.99°
RLA = 147.63°

Kepler Spacecraft Heliocentric Orbit

Earth-trailing heliocentric orbit, a = 1.0133 AU. Observatory trails behind Earth as it orbits Sun and drifts away from us at about 1/10th of 1 AU per year. Period: 372.57 days.

Eccentricity e = 0.036116
Perihelion r_p = 0.97671 AU
Aphelion r_a = 1.0499 AU
Inclination i = 0.44747°

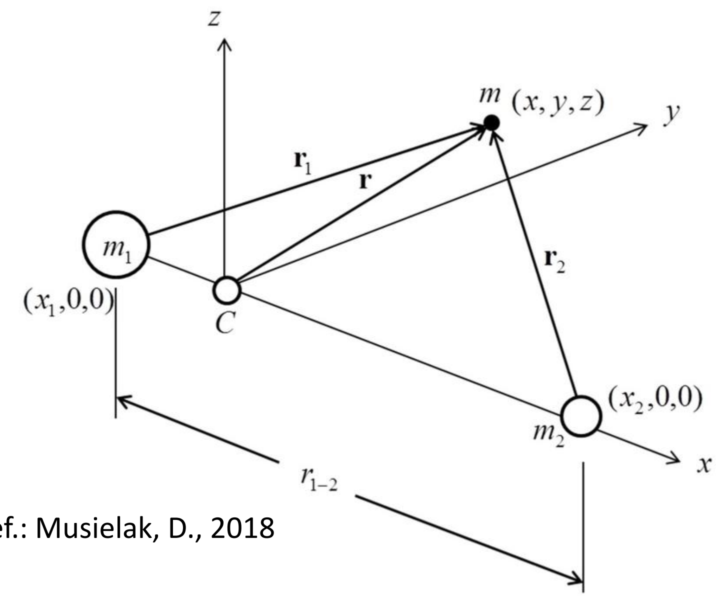


In 2014-2015, Kepler observed Neptune and sent data to study planetary weather! https://svs.gsfc.nasa.gov/4559

NASA image

Restricted 3-Body Problem (R3BP)

Consider motion of a body of mass *m* in a co-moving coordinate system, subject to gravitational field of two massive bodies m_1 and m_2 (primary bodies).



Ref.: Musielak, D., 2018

If mass m is small such that $m \ll m_1$, and $m \ll m_2$, it has no effect on motion of primaries.

$$\ddot{\mathbf{r}} = -\frac{\mu_1}{r_1^3}\mathbf{r}_1 - \frac{\mu_2}{r_2^3}\mathbf{r}_2$$

$$\ddot{x} - 2\omega\dot{y} - \omega^2 x = -\frac{\mu_1}{r_1^3}(x + M_2 r_{12}) - \frac{\mu_2}{r_2^3}(x - M_1 r_{12})$$

$$\ddot{y} - 2\omega\dot{x} - \omega^2 y = -\frac{\mu_1}{r_1^3}y - \frac{\mu_2}{r_2^3}y$$

$$\ddot{z} = -\frac{\mu_1}{r_1^3}z - \frac{\mu_2}{r_2^3}z$$

$$\mu_1 = Gm_1, \qquad \mu_2 = Gm_2$$

$$M_1 = \frac{m_2}{m_1 + m_2}, \qquad M_2 = \frac{m_1}{m_1 + m_2}$$

We use R3BP to represent motion of a spacecraft in Sun-Earth-Moon system. We can account for effect of perturbing forces such as drag, thrust, magnetic fields, solar radiation pressure, solar wind, nonspherical shapes, etc. Spacecraft TESS orbit represents solution of a R3BP.

TESS: Exoplanet Survey Mission

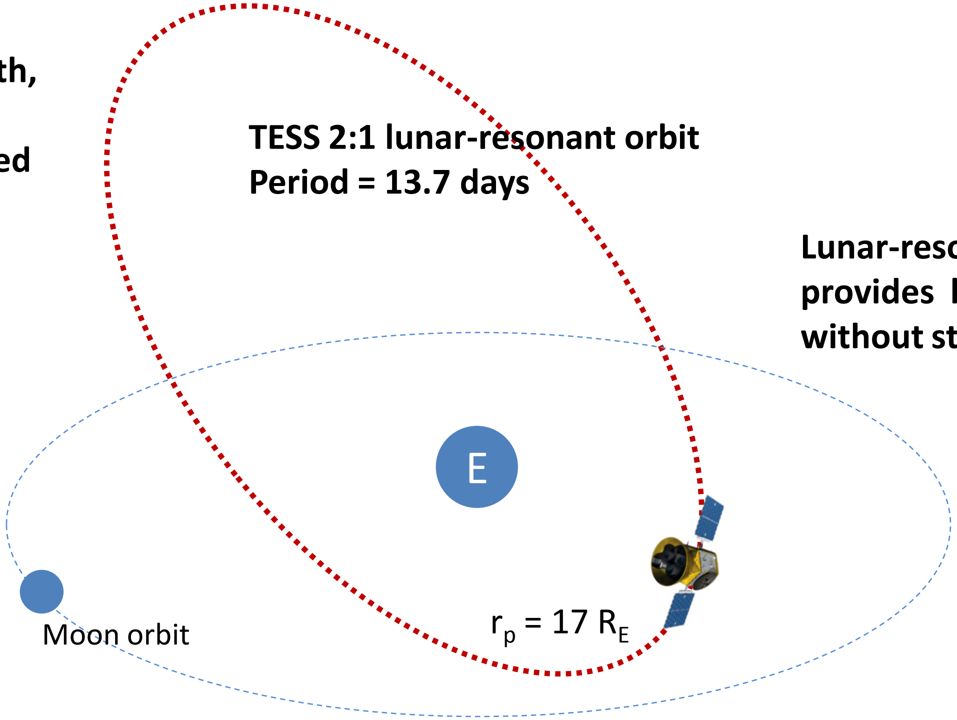
Launched 18 April 2018, TESS has four identical cameras to survey nearly entire celestial sphere in 26 four-week increments over its two-year primary mission. TESS is observing sectors 24°x96° in southern ecliptic hemisphere (first year) and will observe sectors in northern ecliptic hemisphere in second year.

TESS moves on elliptical HEO, with perigee and apogee distances of 17 and 59 R_E.

TESS is in a 2:1 resonance with Moon.
TESS makes two revolutions around Earth, and Moon makes one.
A perturbation to TESS's orbit is corrected by Moon attraction.

Orbit-Orbit Resonances (Mean-motion or Laplace Resonance): Resonance occurs when orbiting bodies exert a regular, periodic gravitational influence on each other, usually because their orbital periods are related by a ratio of small integers.

$$n \equiv \frac{2\pi}{T} = \left(\frac{\mu}{a^3}\right)^{1/2}$$



Lunar-resonant orbit provides long view periods without station-keeping.

TESS Insertion Orbit

TESS Initial Orbit Elements

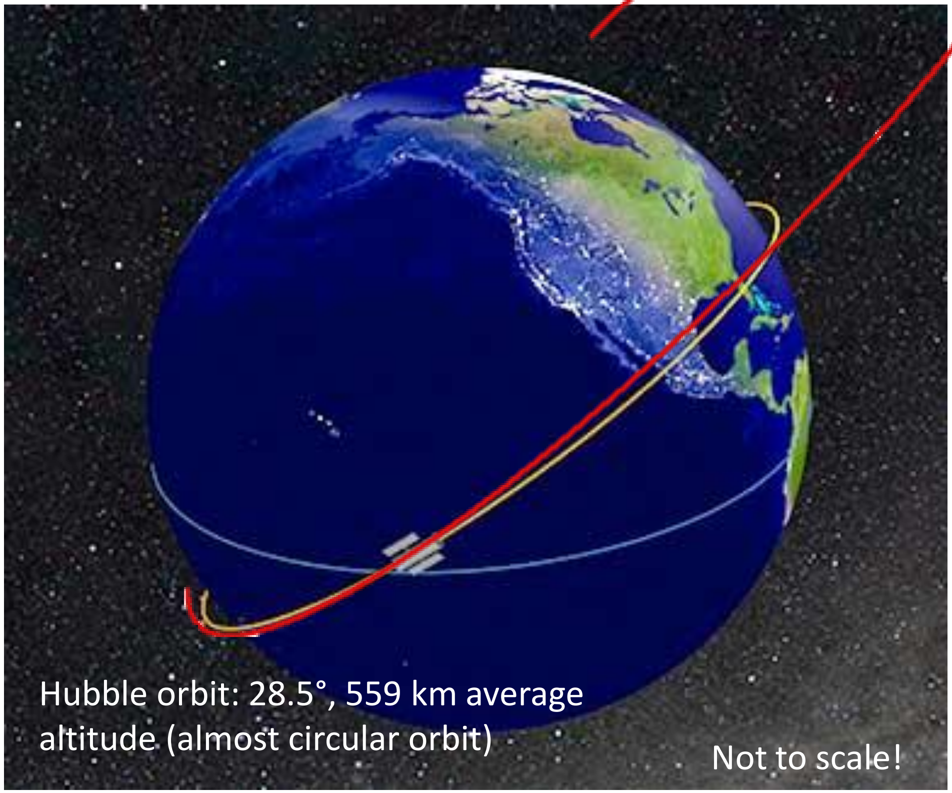
$$r_p = 600 \text{ km}$$

$$r_a = 270{,}000 \text{ km}$$

$$i = 28.5°$$

$$e = \frac{r_a - r_p}{r_a + r_p} = 0.9955$$

i = angle between equatorial plane and plane of orbiting satellite.



Launching at low angles minimizes amount of rocket thrust needed to achieve orbit. *i* = 28.5°, easiest inclination from Cape Canaveral.

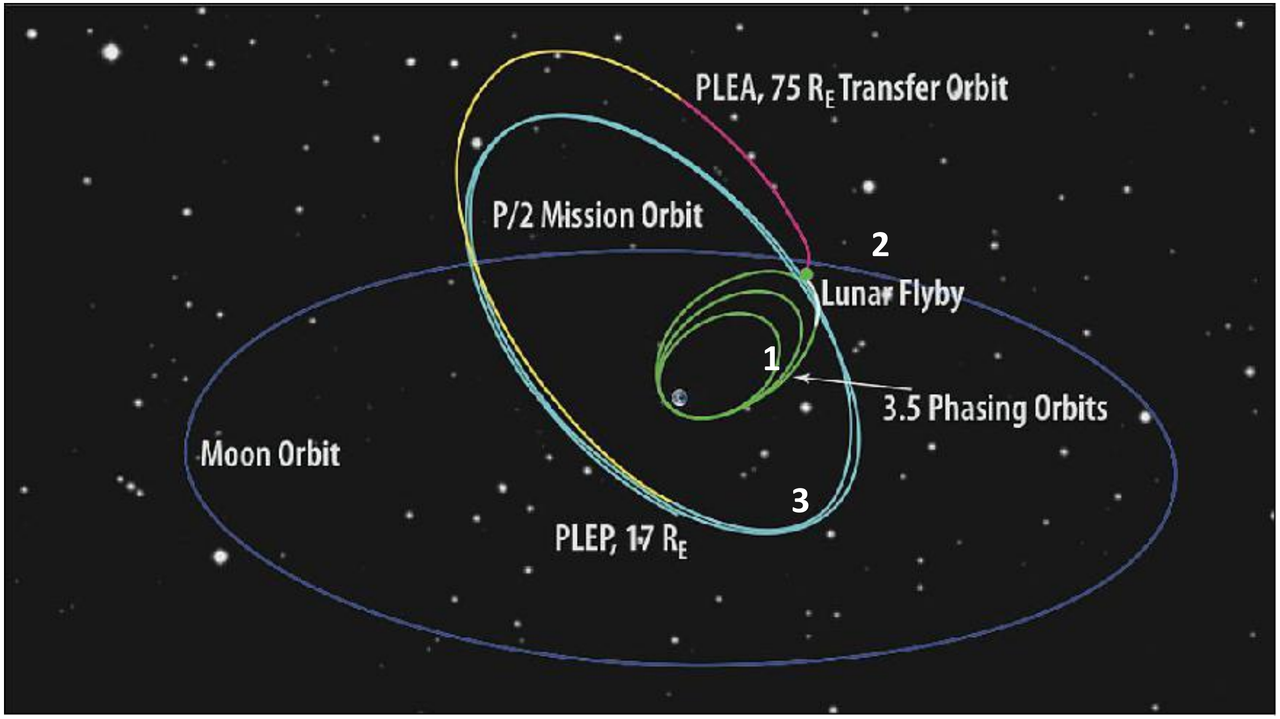
We can see ISS from more places on Earth than the HST due to its highly inclined geocentric orbit and lower altitude. Hubble completes an orbit around Earth about once every 90 minutes.

TESS Orbit Trajectory Cosmic Dance

r_p = 120,000 km
r_a = 400,000 km
a = 240,000 km

$r_p = 17\ R_E$
$r_a = 59\ R_E$
$i = 37°$
t = 13.67 days
(2:1 lunar resonance)

PLEA = post-lunar-encounter apogee.
PLEP = post-lunar-encounter perigee



Ref.: Parker, et al., 2018

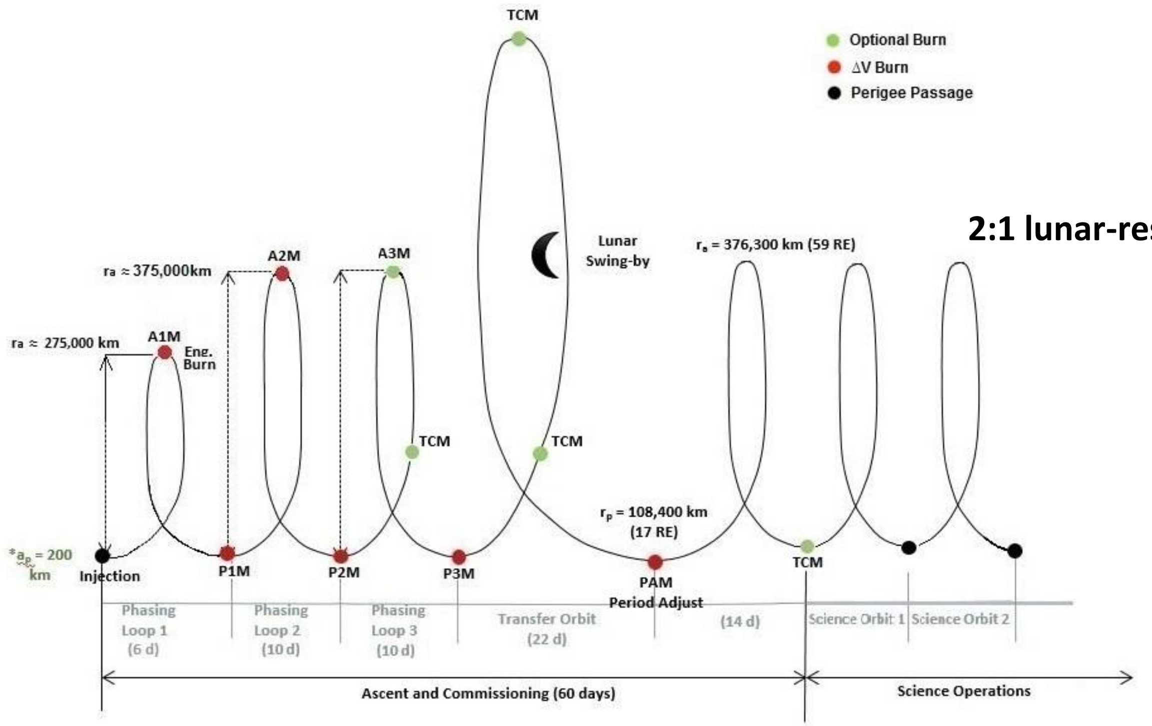
(1) After launch orbit insertion, TESS made 3.5 phasing loops to raise apogee to lunar distance.
(2) Had lunar flyby to increase perigee radius and inclination to their desired values.
Gravitational assistance from Moon changed its orbital inclination to 37° from ecliptic plane.
(3) Performed large transfer orbit with final maneuver to establish resonance.
TESS used its propulsion to make corrections to achieve its final orbit

https://directory.eoportal.org/web/eoportal/satellite-missions/t/tess

TESS Orbit Design

- **TESS executed a two-month commissioning phase to put spacecraft successfully into its target mission orbit, reached efficiently using a small propulsion system (Δv ~3 km/s) augmented by a lunar gravity assist.**
- **Stable resonance in CR3BP refers to tendency of a closed orbit to exhibit long-term stability without station-keeping maneuvers when its period is a ratio of that of a perturbing body, and proper phasing is established ➔ Kozai-Lidov mechanism.**

2:1 lunar-resonant orbit

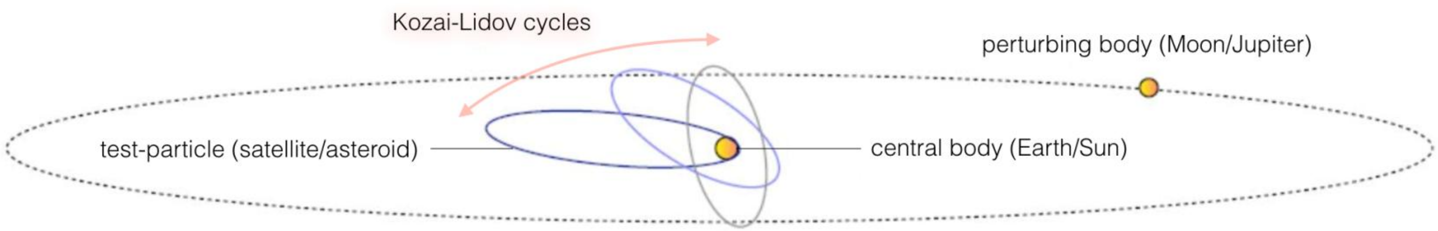


Ref.: Parker, et al., 2018

Period Adjust Maneuver (PAM)

Kosai-Lidov Mechanism

Kozai-Lidov mechanism describes long-term behavior of a highly-eccentric, highly-inclined orbit subject to a third-body perturbing force.



- **Dynamical phenomenon affecting orbit of a binary system perturbed by a distant third body under certain conditions, causing orbit's argument of pericenter to oscillate about a constant value ➔ it leads to a periodic exchange between its eccentricity and inclination.**

- **Process occurs on timescales much longer than orbital periods. It can drive an initially near-circular orbit to arbitrarily high eccentricity, and flip an initially moderately inclined orbit between a prograde and a retrograde motion.***

Kosai mechanism timescale:

$$T_{\text{Kozai}} = 2\pi \frac{\sqrt{GM}}{Gm_2} \frac{a_2^3}{a^{3/2}} \left(1 - e_2^2\right)^{3/2} = \frac{M}{m_2} \frac{P_2^2}{P} \left(1 - e_2^2\right)^{3/2}$$

* Lidov-Kozai mechanism, combined with tidal friction, produce Hot Jupiters (gas giant exoplanets orbiting their stars on tight orbits)

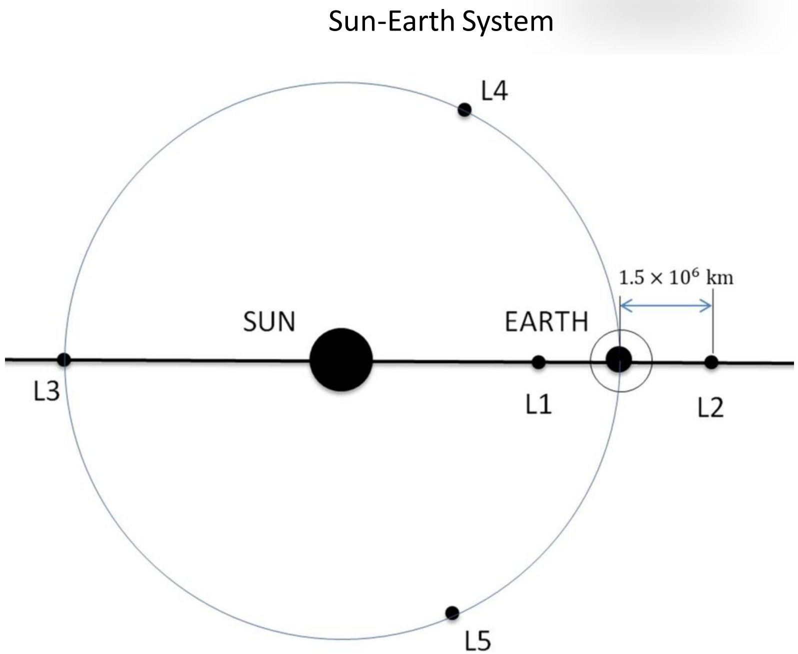
Euler-Lagrange Equilibrium Points

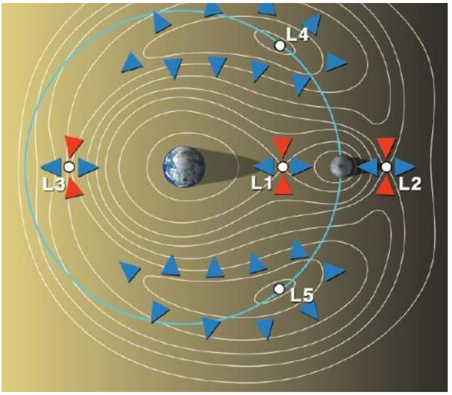
Equilibrium Points: regions near two large bodies m_1 and m_2 in orbit where a smaller object will maintain its position relative to large orbiting bodies. At these positions combined gravitational pull of two large bodies provides precisely centrifugal force required by small body to rotate with them.

At other locations, a small object would go into its own orbit around one of large bodies, but at equilibrium points gravitational forces of two large bodies, centripetal force of orbital motion, and (for certain points) Coriolis acceleration all match up in a way that cause small object to maintain a stable or nearly stable position relative to large bodies

In R3BP system, points L1, L2 and L3 are known as Euler's points, while L4 and L5 are Lagrange's points.

Ref.:, D. Musielak, 2018



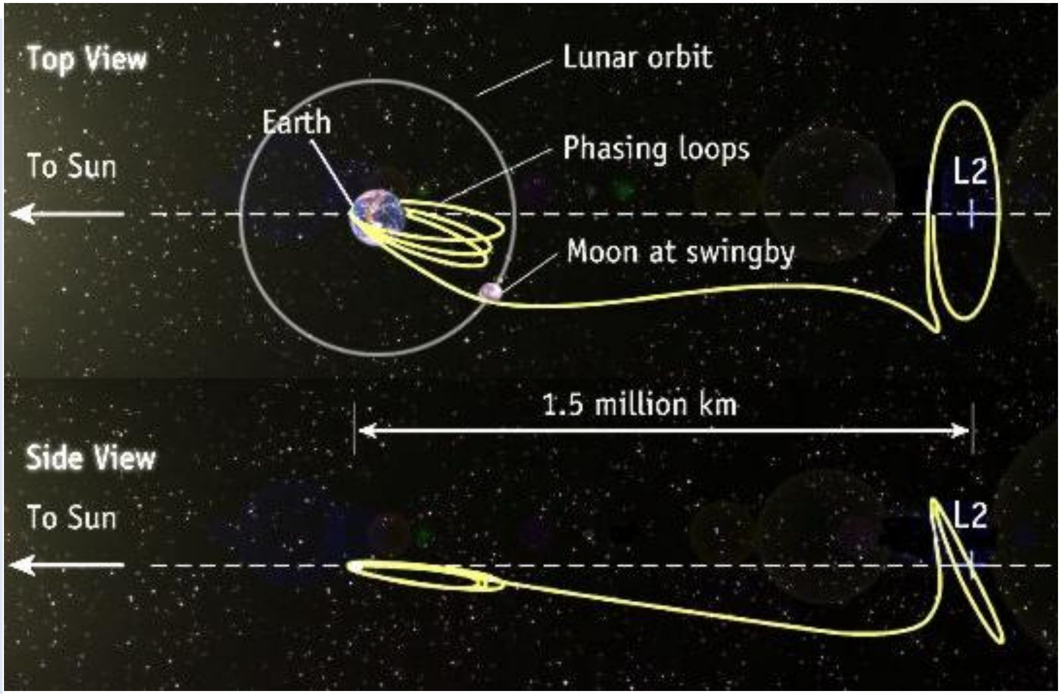


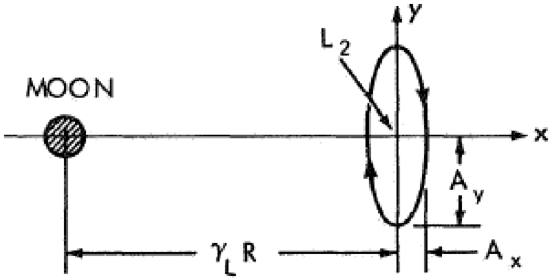
Lissajous/Halo Orbit about SEL2

Lissajous orbit: quasi-periodic orbital trajectory that an object can follow around an equilibrium point.

Halo orbit: periodic, 3-D orbit that results from an interaction between gravitational pull of two primary bodies and Coriolis and centrifugal accelerations on a spacecraft.

In 1968, R. Farquhar advocated using spacecraft in a halo orbit about L2 as a communications relay station for an Apollo mission ➔ spacecraft in halo orbit would be in continuous view of both Earth and lunar far side.

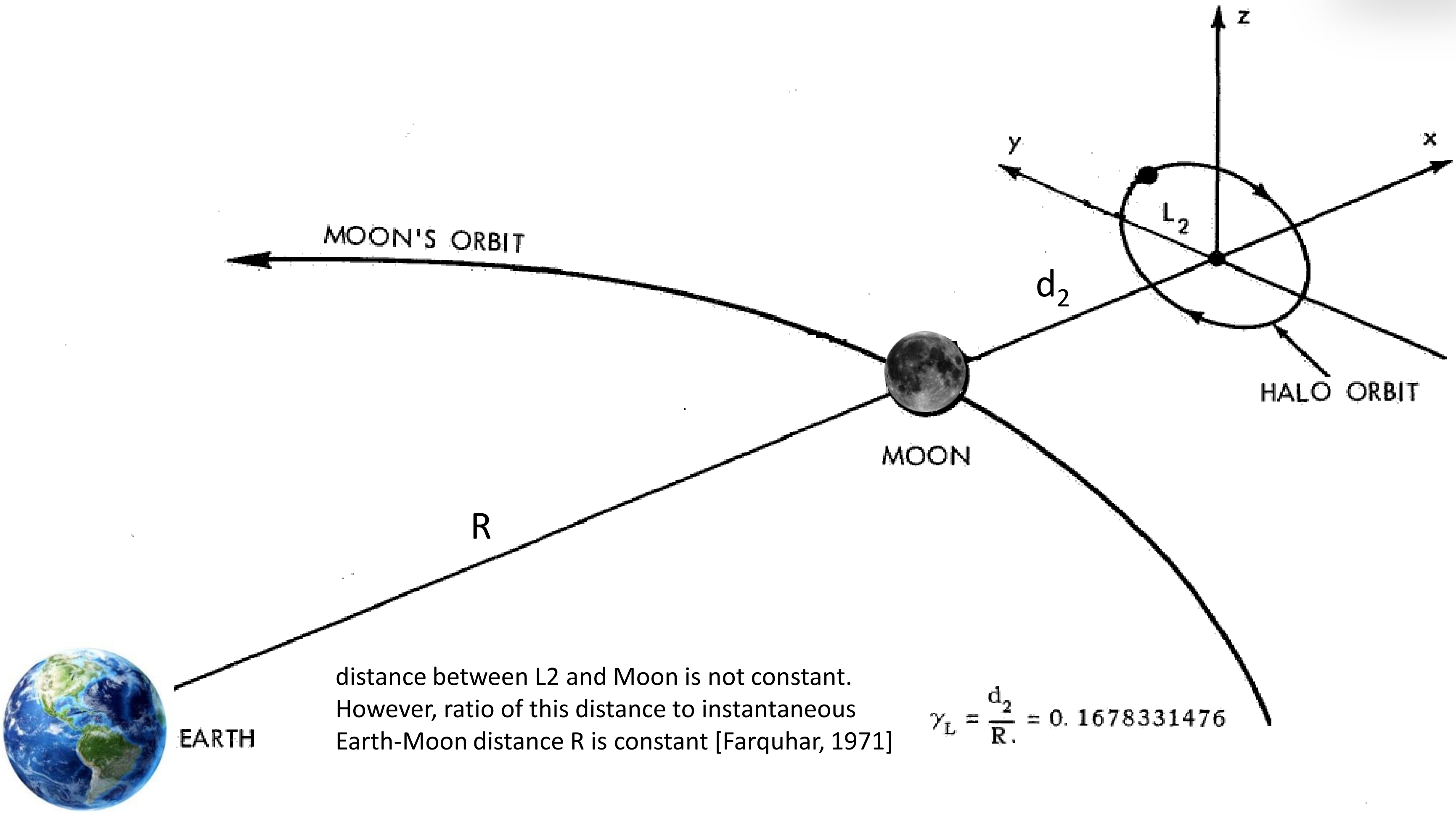




PERIODIC ORBIT AT L_2

Since 1978, various spacecraft have been placed on orbit about L1 and L2 (Sun-Earth and Earth-Moon systems)

L2 Halo Orbit



distance between L2 and Moon is not constant.
However, ratio of this distance to instantaneous
Earth-Moon distance R is constant [Farquhar, 1971]

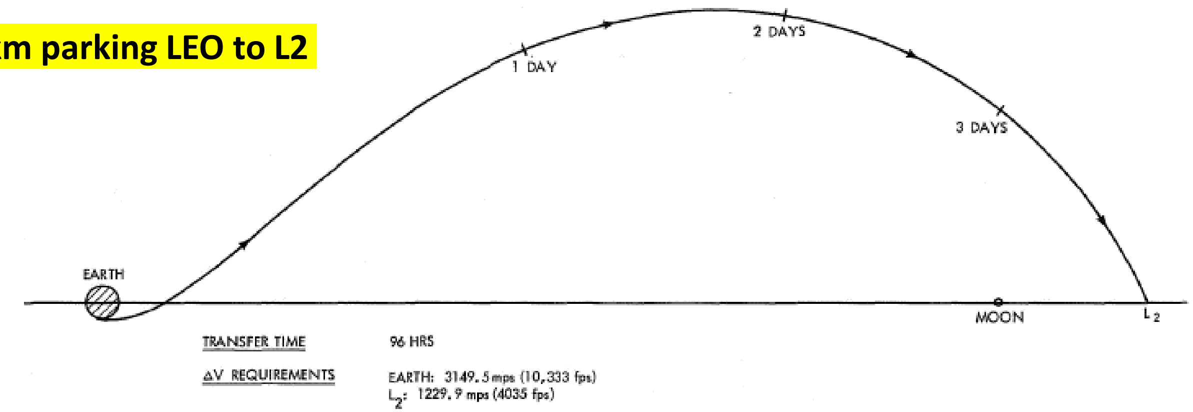
$$\gamma_L = \frac{d_2}{R} = 0.1678331476$$

Ref.: Farquhar, R.W.: The Utilization of Halo Orbits in Advanced Lunar Operations, NASA TN D–6365

Transfer Trajectory to L2

Two-impulse transfer from 185.2 km parking LEO to L2

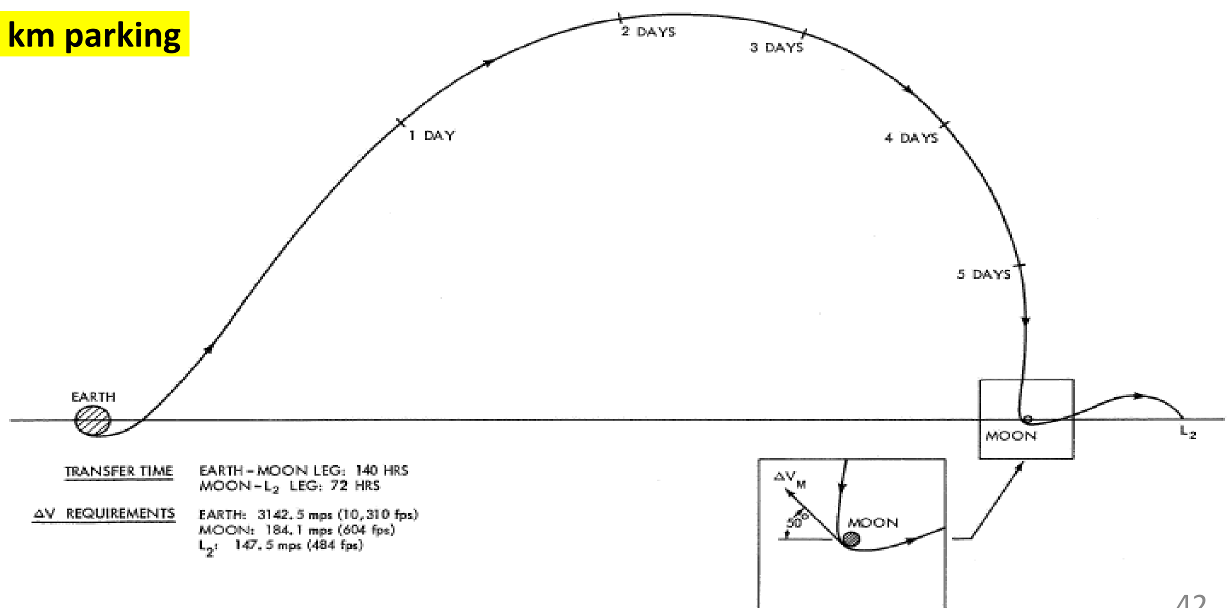
Faster, greater Δv



Three-impulse transfer from 185.2 km parking LEO to L2 (lunar flyby trajectory)

Slower, lower Δv

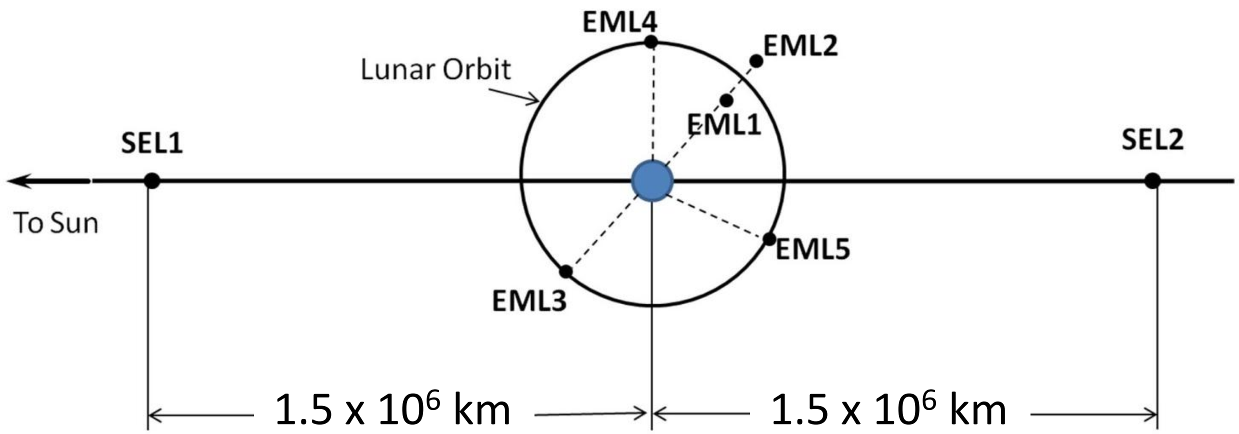
Spacecraft would fly past Moon at a perilune altitude ~100 km for a propulsive gravity slingshot maneuver towards L2, where it'd use its propulsion to enter into a halo orbit.



Ref.: Farquhar, 1971

Equilibrium Points in Earth's SOI

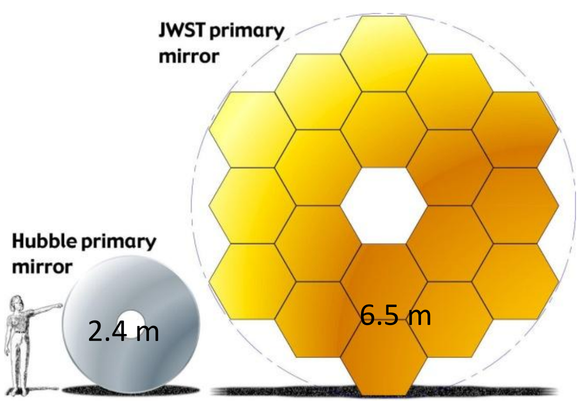
In Sun-Earth-Moon system, there are **seven equilibrium points in Earth's SOI**. Five points result from Earth-Moon System, denoted as EML1, EML2, EML3, EML4, and ELM5, and two points belong to Sun-Earth System, SEL1 and SEL2. In reference frame depicted, Sun-Earth line is fixed, and Earth-Moon configuration rotates around Earth.

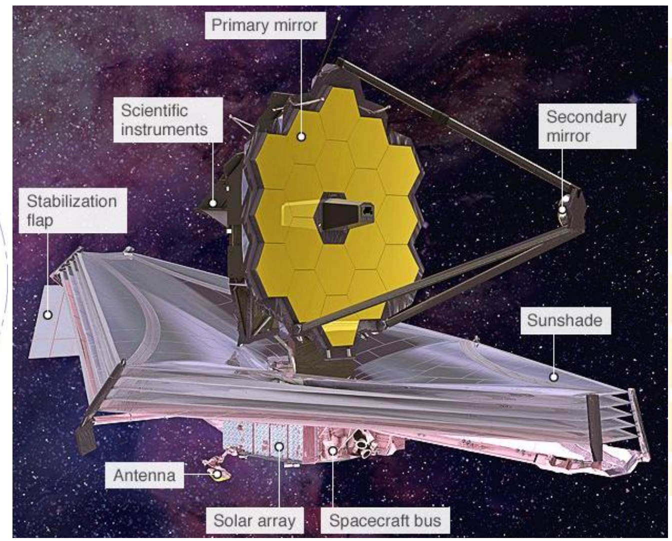


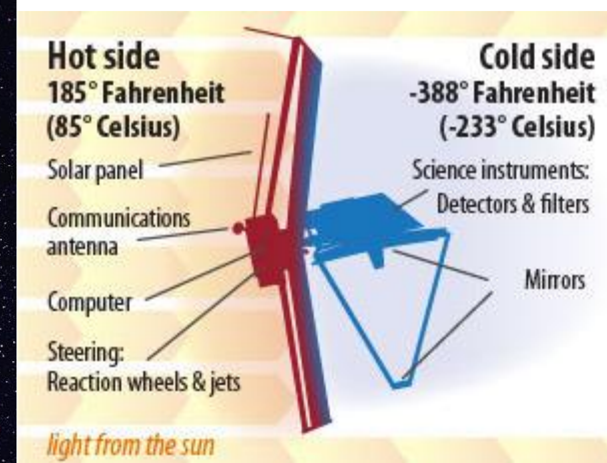
SEL1 suited for making observations of Sun–Earth system. Spacecraft currently orbiting there include Solar and Heliospheric Observatory (SOHO) and Advanced Composition Explorer (ACE).
SEL2 is a good spot for space-based observatories.
EML1 allows easy access to Lunar and Earth orbits with minimal change in velocity.
EML2 currently used by Chinese spacecraft in lunar exploration mission.

James Webb Space Telescope (JWST)

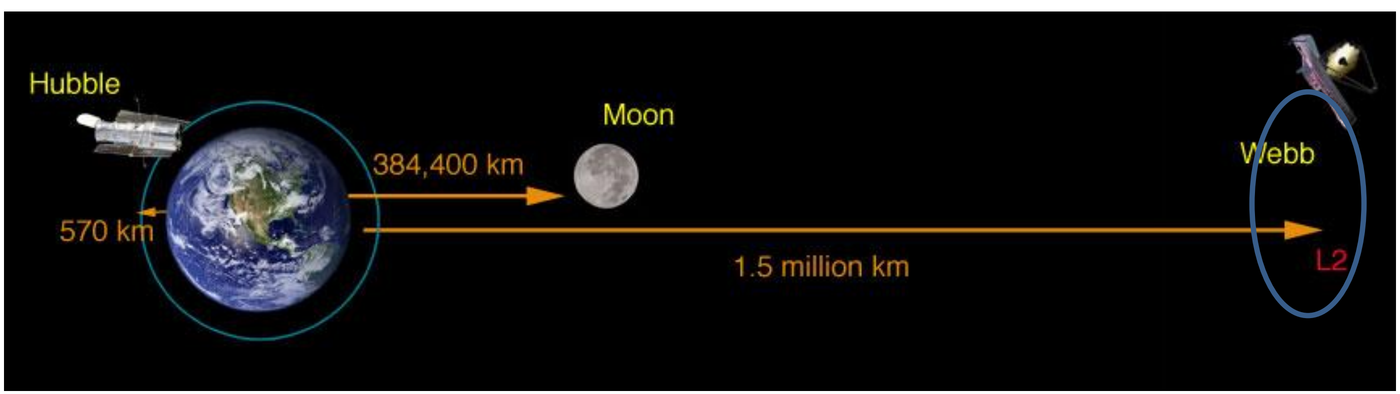
JWST will orbit SEL2 to allow telescope stay in line with Earth as it moves around Sun. It allows large sunshield to protect telescope from light and heat emanating from Sun and Earth (and Moon).







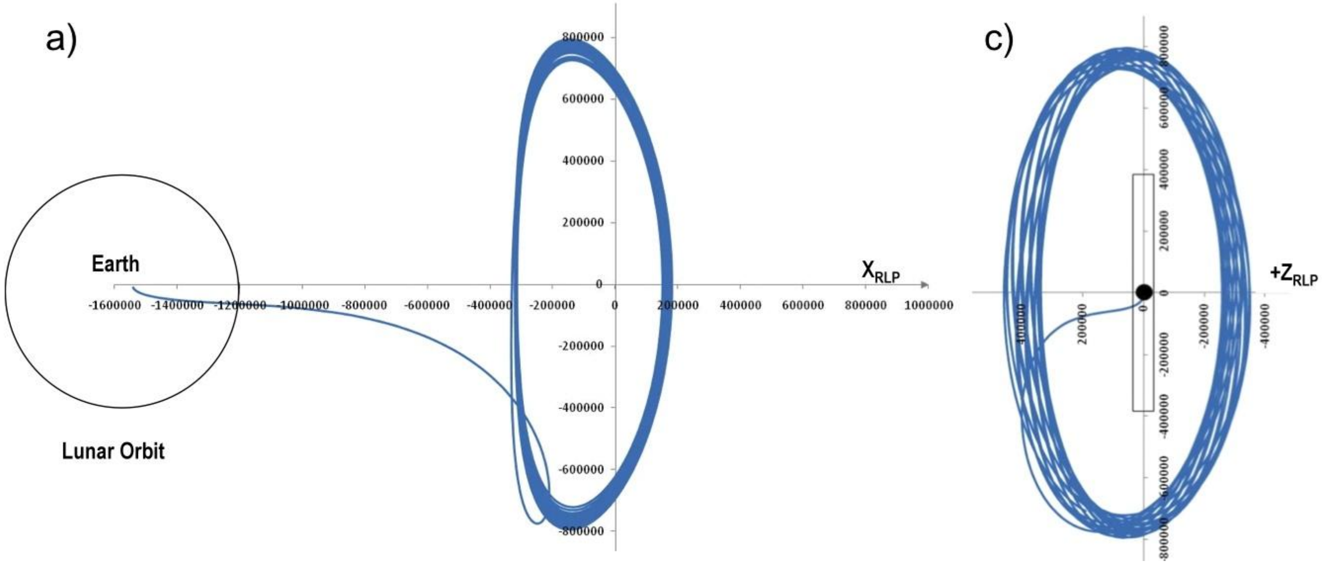
To SUN



JWST Trajectory and Orbit about SEL2

JWST will orbit around SEL2 point (1.5 million km from Earth, i.e., four times Earth-Moon distance).

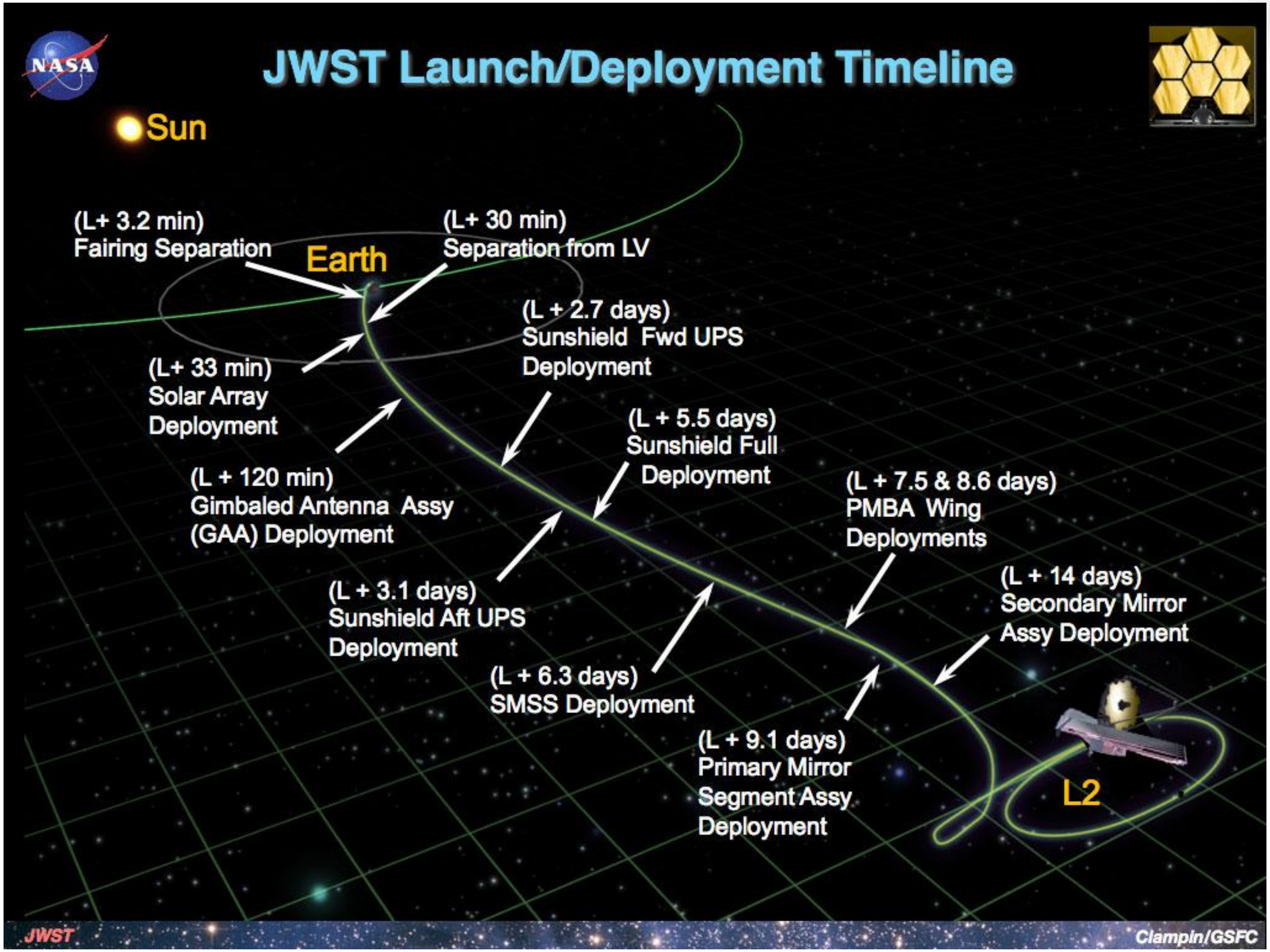
JWST distance from L2 varies between 250,000 to 832,000 km. Orbit period ~ six months.



Maximum distance from Earth to JWST orbit will be ~ 1.8 million km.
L2 is a saddle point (not stable) in Solar System gravitational potential.
JWST will need to regularly fire onboard thrusters to maintain its orbit around SEL2.
These station-keeping maneuvers will be performed every 21 days.

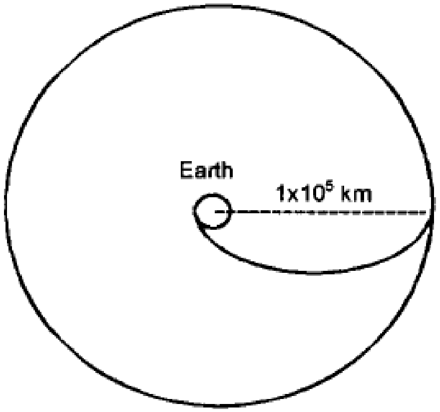
https://jwst-docs.stsci.edu/display/JTI/JWST+Orbit

JWST Launch and Deployment



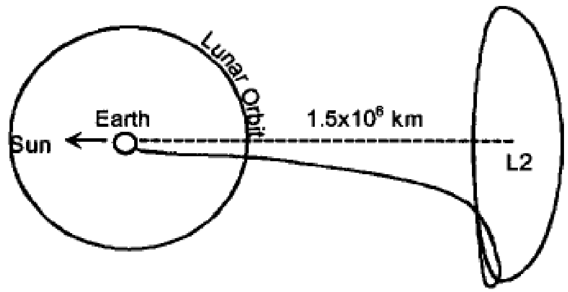
Orbit Comparison

Geocentric HEO Orbit



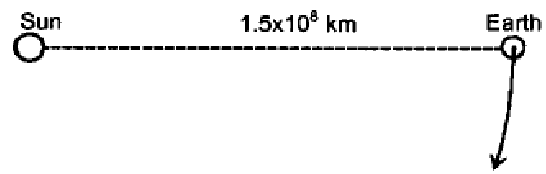
$\Delta V1 = 2.926$ km/sec
$\Delta V2 = 1.280$
Total = 4.206

Heliocentric SEL2 Orbit



$\Delta V = 3.222$ km/sec

Heliocentric Orbit



$\Delta V = 3.267$ km/sec

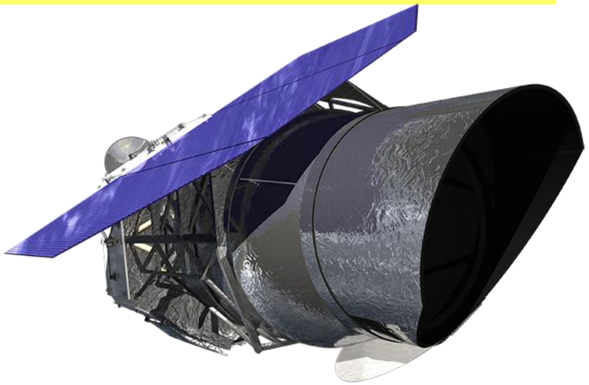
Ref.: Kwok, et al., 2004

WFIRST Quasi-Halo Orbit

Wide Field Infrared Survey Telescope (WFIRST), NASA observatory designed to settle essential questions related to dark energy, exoplanets, and infrared astrophysics.

Current mission design uses an existing 2.4m telescope, which is same size as HST.

WFIRST microlensing survey will detect many more planets than possible with ground telescopes, including smaller mass planets since planet "spike" will be far more likely to be observed from a space-based platform ➔ lead to a statistical census of exoplanets with masses greater than a tenth of Earth's mass from outer habitable zone out to free floating planets

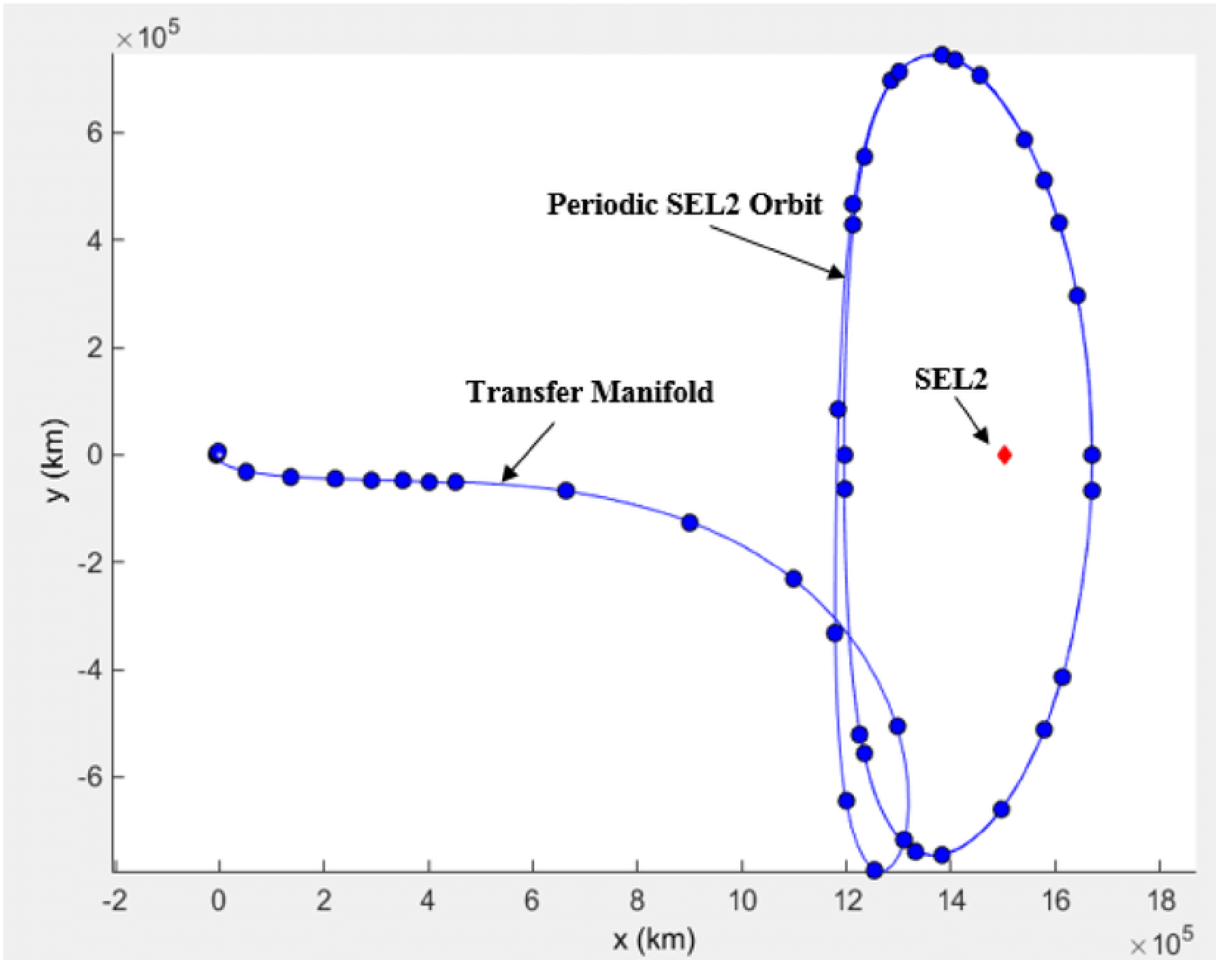


https://svs.gsfc.nasa.gov/cgi-bin/details.cgi?aid=12153

WFIRST observatory will orbit SEL2. Not a halo orbit, as it doesn't pass through exact same points every 6 months. Orbit is evolving and opening in a quasi-periodic fashion, and hence is called a Quasi-Halo.

NASA's Goddard Space Flight Center/Wiessinger

WFIRST at SEL2



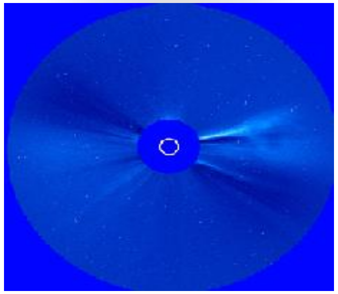
Ref.: Farres, et al., *High fidelity modeling of SRP and its effect on the relative motion of Starshade and WFIRST*, 2018.

WFIRST and Starshade

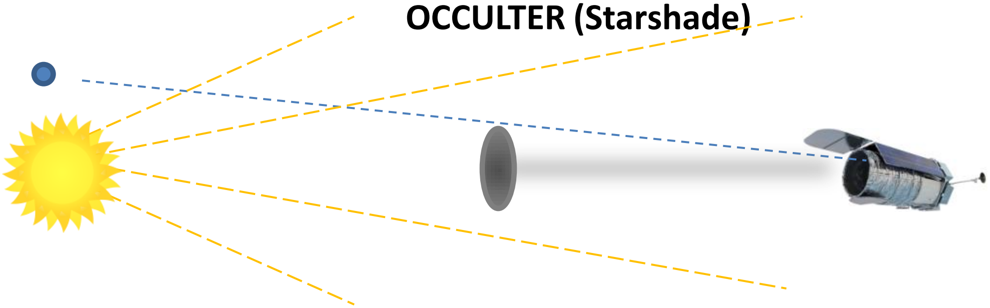
• WFIRST will use a 2.4 meter mirror with Wide-Field and Coronagraph Instruments to achieve its mission objectives (Coronograph to search for exoplanets)
• Using a Starshade would make possible to detect Earth-sized planets in habitable zones of nearby stars.
• Coronagraph blocks Sun's bright disk, allowing much fainter corona to be seen.
• Coronagraph is telescope fitted with lenses and occulting shields inside telescope body.
• Occulter is a shield outside telescope that blocks light of a star in order to view fainter bodies beside it.
• Occulter or starshade is flown in conjunction with a space telescope. Once star, starshade, and telescope are aligned, occulter blocks star's light, so telescope can view faint planets orbiting star.

CORONAGRAPH

Image of a coronal mass ejection, observed by Solar Terrestrial Relations Observatory (STEREO). White circle indicates solar disk location and size.

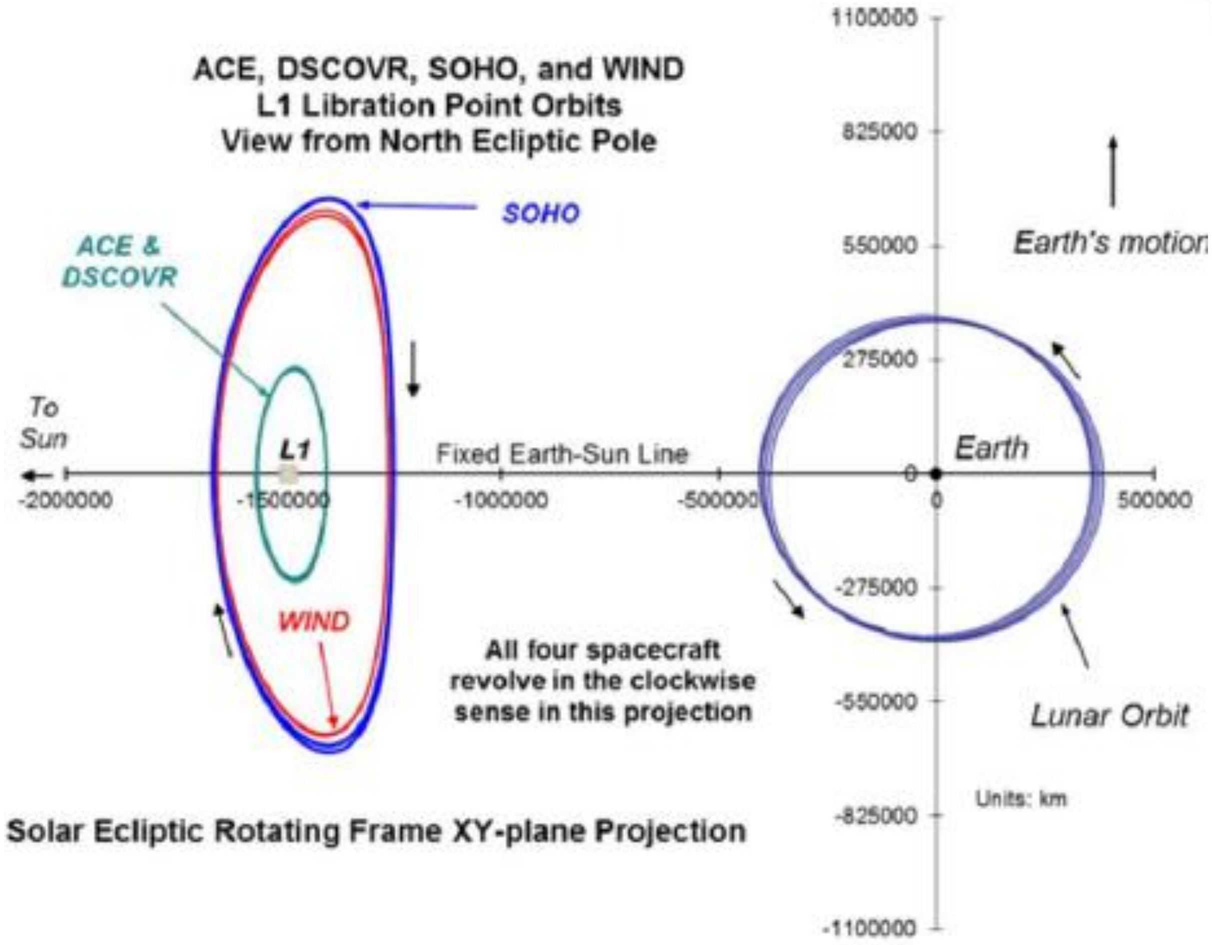


OCCULTER (Starshade)



A properly placed and shaped occulter will cast a shadow over telescope while letting light reflected from planet to pass unimpeded.

Halo Orbits about SEL1



Ref. Roberts, et al., 2015

Farside of the Moon

NASA's Lunar Reconnaissance Orbiter (LRO) has greatly improved our understanding of the Moon, orbiting about it since 2009. Spacecraft has collected hundreds of terabytes of data that allow scientists to create extremely detailed maps of Moon's topography.
On 18 June 2009, Atlas V vehicle launched LRO. It now orbits the Moon on a circular orbit at 50 km altitude.


NASA Visualization Explorer, https://svs.gsfc.nasa.gov/11747

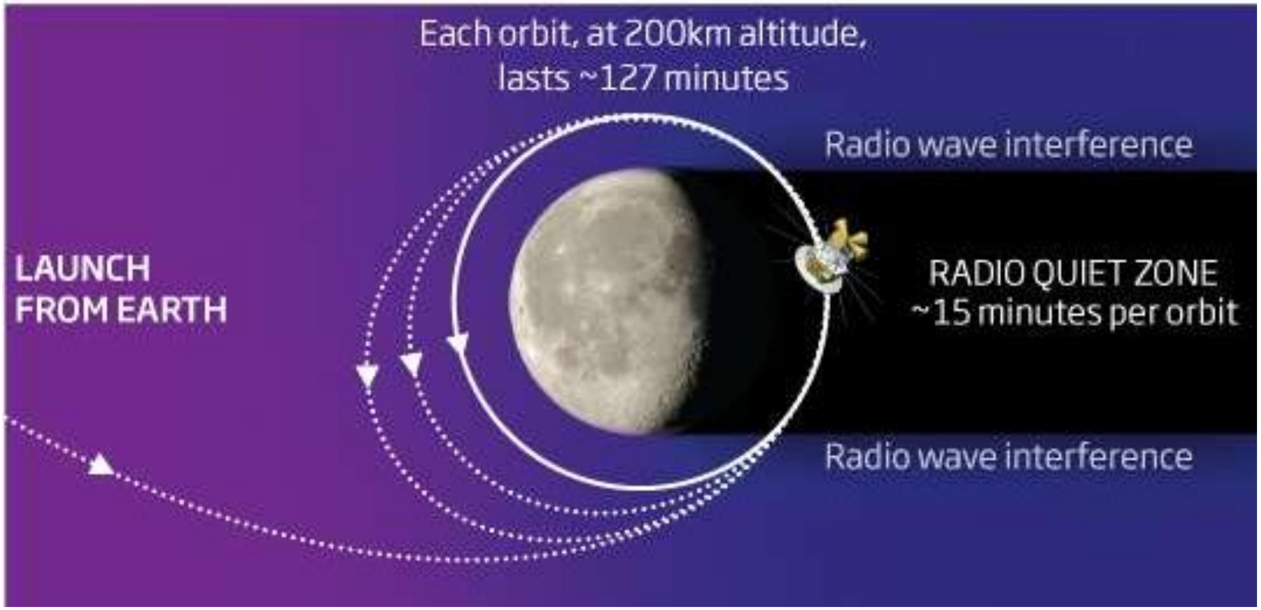
New **mission proposed to study universe 80 to 420 million years after birth** (cosmic Dark Ages).
Observations require quiet radio conditions, shielded from Sun and Earth EM emissions.
This zone is only available on lunar far side.
Other space exploration missions are also considered in this part of Earth-Moon system.

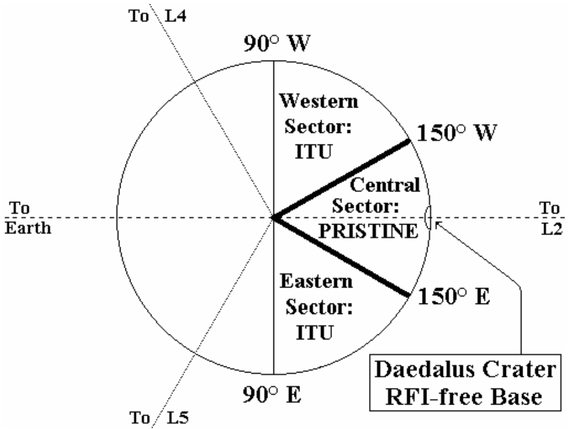

Lunar Reconnaissance Orbiter

Dark Ages Radio Explorer (DARE)

Goal of Dark Ages Radio Explorer (DARE): to study universe 80 to 420 million years after birth.

To maximize time in best science conditions, DARE spacecraft requires low, equatorial lunar orbit.





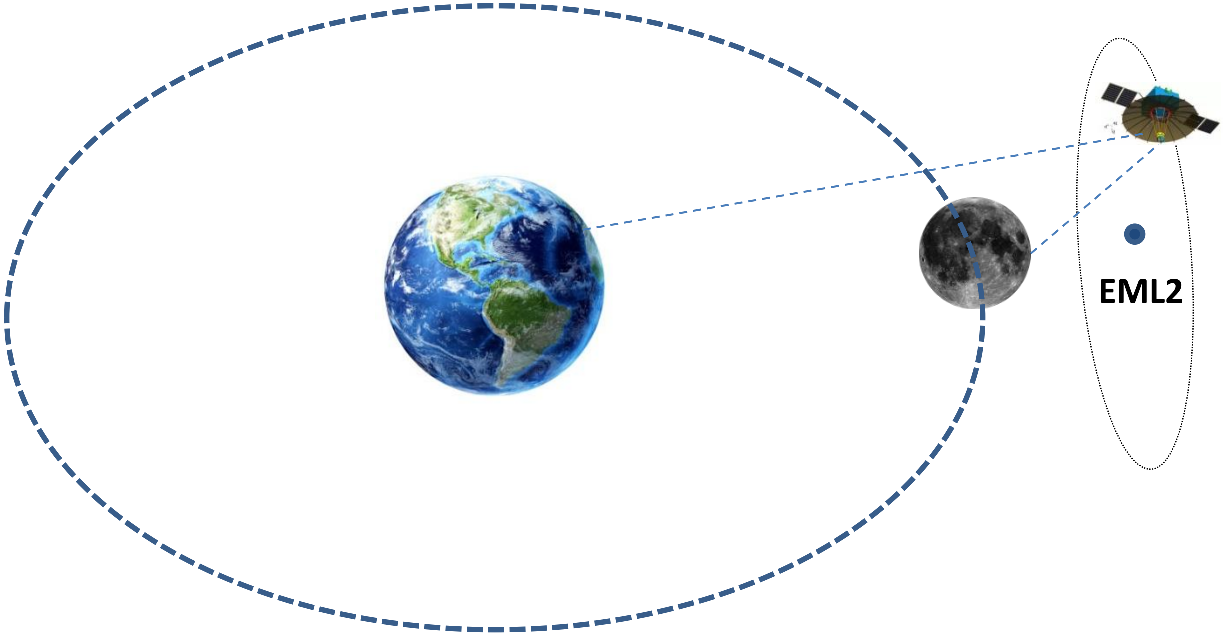
Ref.: Plice and Galal, 2017.

Image taken from Bang D. Nhan's presentation, https://slideplayer.com/slide/13909736/

More on DARE mission at https://www.colorado.edu/dark-ages-radio-explorer/dark-ages-radio-experiment-dare

Queqiao Spacecraft Orbit about EML2

In May 2018, China launched Queqiao, a communications relay spacecraft to support Chang'e 4 lunar exploration mission. Queqiao now orbits about Earth-Moon L2 point. A robotic lander and a rover named Yutu 2 launched on 7 December and entered lunar orbit on 12 December 2018.



Orbit Design Approach

Parametric Analysis

Performed to address impact of variables on required ideal orbit. Preliminary analysis provides an initial solution (guess) for all parameters that characterize feasible orbit.

Orbit Optimization

Initial guess solution (represented by appropriate orbit vector) is refined to minimize Δv required to leave the launch vehicle while at the same time meeting trajectory constraints. Initial condition for direct transfer trajectory is characterized by launcher parking orbit (e.g., 300 km altitude, *i* = 35°). Optimization process carried out using sophisticated algorithms .

Spacecraft Orbit Design Tools

[General Mission Analysis Tool (GMAT)]
[Copernicus]
[Mission Analysis Low-Thrust Optimization program (MALTO)]
Orbit Determination Tool Kit (ODTK), version 6.3.2. http://agi.com (Analytical Graphics, Inc.)
Goddard Trajectory Determination System (GTDS). (NASA Goddard Space Flight Center)

Optimization process minimizes necessary ΔV, required to inject the probe from launch vehicle parking orbit onto escape hyperbola by defining a constrained optimization problem. Orbital parameters of departing hyperbola act as optimization variables whereas position angle variation and maximum distance attained from Earth is constrained to some proper values.

- **Engineers and scientists have identified non-irregular planetary motion in our Solar System, using numerical models of stability over millions of years.**

- **Space exploration program presents a host of challenging problems (3BP/4BP) that rocket scientists and engineers help every day to resolve. But many problems remain. For example, we need to understand motion of as yet to be designed space robots, solve motion of interplanetary probes that can interact with planetary bodies, and determine spacecraft orbital maneuvers required for missions to Moon and across Solar System. All that and much more requires highly sophisticated analytical tools and new interdisciplinary efforts by all of us.**

Let’s us uncover the secrets of the heavens … *Ad Adstra*!

References

- Musielak, D.E., *Euler Celestial Analysis*, with Introduction to Spacecraft Orbits." ISBN 978-1-5462-3186-8, 2018.
- Plice, L. and Galal, K., "DARE Mission Design: Low RFI Observations from a Low-altitude, Frozen Lunar Orbit." AAS 17-333, 2017 AAS/AIAA Space Flight Mechanics Meeting, San Antonio, TX, Feb. 5 – 9, 2017.
- Farquhar, R., "The Utilization of Halo Orbits in Advanced Lunar Operations."NASA TN D-6365, July 1971
- Parker, J. J. K., Lebois, R. L., Lutz, S., Nickel, C., Ferrant, K., and Michaels, A., "Transiting Exoplanet Survey Satellite (TESS) Flight Dynamics Commissioning Results and Experiences." AAS 18-408, 2018
- Kwok, J. H., Garcia, Mark D., Bonfiglio, E., and Long, S. M., "Spitzer Space Telescope mission design." Proceedings Volume 5487, Optical, Infrared, and Millimeter Space Telescopes; (2004)
- Roberts, C., Case, S., and Reagoso, J., "Lissajous Orbit Control for the Deep Space Climate Observatory Sun-Earth L1 Libration Point Mission." 2015 AAS/AIAA Astrodynamics Spacialist Conference, Paper ASC 15-611.
- Sergeyevsky, A. B., Snyder, G. C., and Cunniff, R.A., "Interplanetary Mission Design Handbook, Volume I, Part 2, Earth to Mars Ballistic Mission Opportunities, 1990-2005." JPL PUBLICATION 82-43, September 1983.
- Farres, A., Webster, C., and Folta, D., "High fidelity modeling of SRP and its effect on the relative motion of Starshade and WFIRST," 2018 Space Flight Mechanics Meeting, AIAA SciTech Forum, Paper AIAA 2018-2227.

Keplerian Orbit Definition (2-Body Motion)

$$r = \frac{a(1-e^2)}{1 + e\cos\theta}$$

$$e = (r_a - r_p)/(r_a + r_p)$$

r = Radius from spacecraft CM to central body center

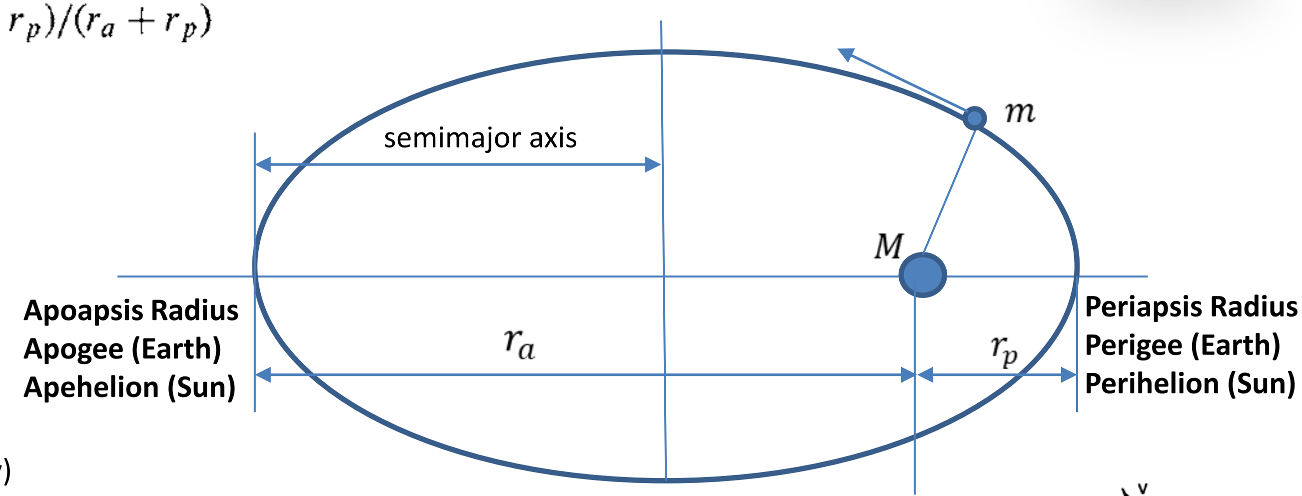
θ = true anomaly

e = eccentricity

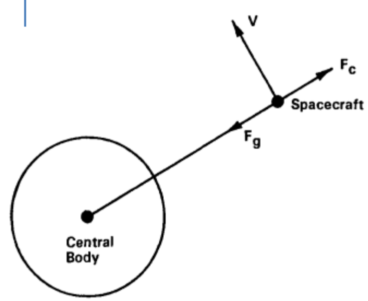
$$r = R_0 + h$$

h = Spacecraft altitude

R_0 = mean equatorial radius (central body)



Motion of a spacecraft in Solar System is dominated by one central body at a time:
1) Spacecraft motion governed by attraction to a single central body (*M*)
2) Spacecraft mass (*m*) is negligible compared to that of central body, *m* << *M*
3) Bodies are spherically symmetric with masses concentrated at their centers.
4) No forces act on bodies except for gravitational forces and centrifugal forces acting along line of centers.



Forces on spacecraft

centrifugal $F_c = mV^2/r$

gravitational $F_g = MmG/r^2$