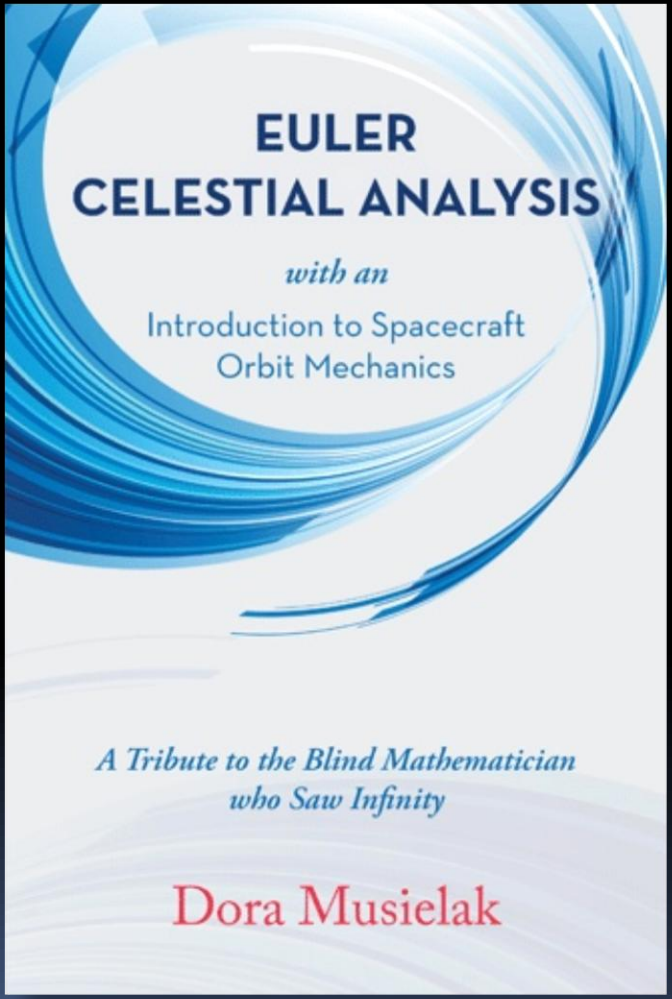
EULER CELESTIAL ANALYSIS

with an Introduction to Spacecraft Orbit Mechanics



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Rocket Scientist, Researcher, Educator, Author

A book to pay homage to Leonhard Euler for his contributions to mathematical astronomy.

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EULER CELESTIAL ANALYSIS

A new, authentic and authoritative book written to highlight mathematician Leonhard Euler's contributions to analytical astronomy

- Fully-researched, with links to Euler's major memoirs, essays, and correspondence.
- Description of major scientific discoveries such as comets and solar eclipses, and Euler's pioneering analysis of the three-body problem.
- Rich in technical detail, connecting Euler's mathematical astronomy with the work of other mathematicians and astronomers in other times and places.
- Euler's social, cultural, and educational legacies.
- Includes little known facts about Euler.
- The book includes a special chapter on orbital mechanics to prepare readers for advanced study in celestial mechanics.

VISION

The "so what"? factor

To offer an elegant and unbiased portrait of a remarkable mathematician, Dora Musielak uses his works to explore how Euler built the foundation for the rigorous study of motion in our Solar System.

With his exquisite flair for analysis, Euler stated the three-body problem of celestial mechanics, and he derived the differential equations for the general n-body problem, identifying all the integrals of motion. He studied comets, eclipses, derived planetary orbits, and pioneered the study of planetary perturbations. Old and blind, Euler put forward the most advanced lunar theory of his time.

Euler Celestial Analysis also provides an introduction to spacecraft orbit mechanics, a branch of celestial mechanics that studies spaceflight and that has revolutionized the direct exploration of the heavens.

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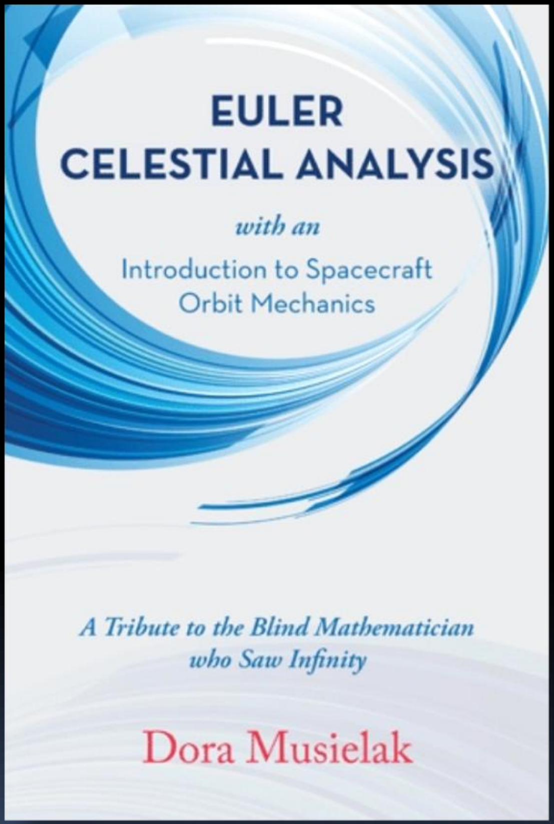

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Under the Prussian Sky

Sidera quod tantis cieant se viribus aequis
In motu terrae plurima signa docent. Euler

At thirty-four, Euler had a universe of ideas that would illuminate the Prussian Academy in the following decades. During his first years in Berlin, Euler published his first book of astronomy and dozens of articles related to his research in celestial mechanics. He also established a new branch of mathematics (calculus of variations), wrote the book that lays the foundations of modern mathematical analysis, and derived new methods to tackle many problems in mathematical physics. Despite his partial blindness, Euler once again became involved with observational astronomy.

The Berlin Observatory was not in the same class as those in Paris or England, not even comparable to the St. Petersburg Observatory. In 1741, its director was Johann Wilhelm Wagner, a rather unknown German astronomer. Initially, when Leibniz proposed the establishment of the Academy as a society for scholars, he conceived the observatory linked to the building project. The plan was to erect the new observatory over its gatehouse on Unter den Linden, Berlin's new and splendid boulevard. To finance the sciences, the privilege of the Royal Prussian Society was to compile, publish and sell the Brandenburg-Prussian almanacs.

The first director of the observatory, from 1700 to 1710, was Gottfried Kirch. He was a respected and talented German astronomer who had studied astronomy with Erhard Weigel in Jena and Johannes Hevelius in Gdańsk. Since 1667, Kirch had published calendars, and built his telescopes. He became famous for discovering comets and introducing new constellations.

When Kirch arrived, the Berlin Observatory did not exist. Hence, Kirch carried out his observations from various private

houses. His wife Maria Margaretha (née Winkelmann), and his children Christfried and Christine, also accomplished astronomers, helped Gottfried Kirch with duties in the observatory. Maria Margaretha Kirch, who had acquired vast knowledge of astronomy through self-study, discovered, among other things, the comet of 1702. She and her husband taught astronomy to their children.

From 1700 to 1711, an observing tower with three additional floors was added to the north wing of the complex (Fig. 17). On 15 January 1711, the Royal Prussian Society of Sciences, held its first meeting in the tower, and four days later the observatory was solemnly inaugurated. Gottfried Kirch had died a year earlier. Hence, his assistant Johann Heinrich Hoffmann took the lead.

When Hoffmann passed away in 1716, Christfried Kirch, son of Gottfried, became the Director of the Observatory, while his mother and sister supported his endeavors, especially with the preparation of the almanacs.

Christfried Kirch died in 1740 (his mother was already deceased), leaving the preparation of the almanacs to his sister Christine. An almanac must contain important dates and statistical information such as astronomical data and tide tables. Hence, Christine had the responsibility of performing the required astronomical calculations, and publishing ephemerides, tables giving the calculated positions of celestial objects at regular intervals throughout a period.

Unpaid and unofficial astronomer Christine Kirch, age 44, was also responsible for keeping the accounts and assisted astronomers in the use of the observatory. She also performed her own observations. But Christine lacked resources, as the observing facility was not well equipped.

When Euler arrived in 1741, he could not help but compare the Berlin observatory with that in St. Petersburg. He wrote to Schumacher praising the Russian institution: "the building in which it is housed is so well adapted to astronomical aims that we are unable to propose a better model in that respect."[70]

[70] Nevskaya, N.I. and K.V. Kholshevnikov, "Euler and the Evolution of Celestial Mechanics," in Euler and Modern Science, eds. N.N. Bogolyubov, G.K. Mikhailov, and A.P. SYushkevich . MAA Vol. IV. p. 284.

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celestial bodies (for him it was comets and the planets in the Solar System).

Euler introduced for the first time the Newtonian equations in the form that we recognize today. For example, he defined the velocity of a body in a three-dimensional inertial coordinate system as given by its three components $\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}$, and the acceleration as $\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2}$. We will expand on this important topic in Chapter 7.

Solar Eclipse in 1748

On 25 July 1748, a solar eclipse darkened the sky over Berlin, and despite his vision problems, Euler built his own camera obscura at home and observed it. His former colleague Delisle—now in Paris— had sent numerous information bulletins (*Avertissements*) to astronomers, encouraging them to observe the solar eclipse and record their impressions. And Euler did, using an apparatus he built in his own house, which was similar to the telescope/camera obscura assembly used by Scheiner (Fig. 8).

In his memoir, Euler stated that, "to observe this Eclipse with more success, and to measure and record all features that it would offer, I prepared in my house a dark room, which looked at the South, and directed a 9-foot telescope towards the Sun. With a hole made at the window, I received the image of the star on a white paper."[79]

Euler continues explaining his observation of the eclipse: "I placed this paper perpendicular to the axis of the telescope at a distance, such that the image of the Sun exactly filled a circle that was drawn there. I set the tube until the image was represented by the most distinct manner on paper, and that one could clearly discern all the spots on the Sun, several of which were visible on its disk. The equipment was constructed in such a way that while the tube continually followed the movement of the Sun, the paper

[79] L. Euler, *Sur l'atmosphère de la Lune prouvée par la dernière éclipse annulaire du Soleil*. (On the atmosphere of the moon as proved by the last annular eclipse of the Sun). Presented to the Berlin Academy on December 5, 1748. In Mémoires de l'académie des sciences de Berlin 4, 1750, pp. 103-121 [E142.]

by a similar movement always maintained the same distance with respect to the tube, so that the image of the Sun remained on the circle drawn on the paper."

Euler describes his experience vividly. Despite having small clouds that frequently covered the Sun, he was able to observe the main phases of the entire annular eclipse. An annular solar eclipse happens when the Moon covers the Sun's center, leaving the Sun's visible outer edges to form an annulus around the Moon typically referred as "ring of fire."

Since his colleague Johann Kies had taken measurements of the solar eclipse with great precision, and made his report to the Academy—Euler noted—, he confined his explanations solely to the things that relate to the subject of his own Memoir. Kies was a German astronomer, and thanks to Euler's recommendation, was also professor of mathematics at the Berlin Academy from 1742 to 1754. After that, Kies became director of the Astronomisches Rechen-Institut in Heidelberg.

In his 1748 memoir,[80] Euler stated: "I only intended to arrive at a more accurate determination of the motion of the Moon and its parallax." Parallax refers to the apparent displacement of an observed object due to a change in the position of the observer, of the observed, or of both. The lunar parallax is an important case. You can see this easily just by alternately blinking your left and right eye while looking at the Moon.

By observing parallax, measuring angles, and using geometry, one can determine distance. In the second century B.C., Greek astronomer Hipparchus used two methods to estimate the distance to the Moon, the first using lunar parallax. He made another attempt during a solar eclipse that was a total eclipse at Syene and a partial eclipse at Alexandria.

According to some historians,[81] the result obtained by Hipparchus was between 62 and 73 Earth radii. Today we know the average distance is about 60 radii, varying by a few Earth radii either way because of the ellipticity of the Moon's orbit.

During this astronomical event in 1748, Euler was not just a casual observer but clearly was working as an astronomer taking

[80] Ibid.

[81] A. Pannekoek, *A History of Astronomy*. Dover Publications. 2 November 2011.

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Euler determined the orbit of this comet and presented his results[125] to the Prussian Academy on 6 September 1742. He calculated the perihelion dates, perihelion distances, and inclination of the comet. In his memoir, he stated: "We consider a comet observed in March of this year [1742] to determine its orbit. The observations of which I plan to use were communicated to me by the renowned Astronomer De Lisle, from the Petropolitano Observatory."[126] The data Delisle sent was rather limited. Yet, Euler's analysis was extensive, calculating all orbit elements by considering the comet having parabolic or elliptical orbits.

Using his enormous power of analysis, Euler derived an important differential equation to determine the relation between the true anomaly of the comet's orbit and the time required to move along its path. Beginning from the Keplerian conic equation, and using his stylish algebraic manipulation of trigonometric functions, Euler obtained the cubic equation for the comet's time of flight between any two points along its parabolic orbit. Euler's cubic equation was in the form, $n/N = \left(t + \frac{1}{3}t^3\right)$, where $t = \tan\left(\frac{1}{2}v\right)$, the angle v is the true anomaly, and the left side of the expression represents the elapsed time of the comet.[127] Euler prepared a rather comprehensive Table to calculate the motion of the comet of 1742, as it moved from perihelion ($v = 0°$) to a position in its parabolic orbit where $v = 180°$.[128]

Let us try a simplified derivation of Euler's cubic equation, in his honor. To describe the motion of the comet, we need a function $r = r(t)$ to specify the comet's position r at each time instant of time t. The function must be a continuous function of the variable t in the entire interval of interest. The astronomical position of any object moving across the sky (ephemerides) is based on $r(t)$.

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[125] Euler, L., *Determinatio orbitae cometae qui mense Martio huius anni 1742 potissimum fuit observatus* (Determination of the motion of a comet which can be observed in March of this year, 1742). Miscellanea Berolinensia 7, 1743, pp. 1-90. Opera Omnia: Series 2, Volume 28, pp. 28 - 104. [E58]

[126] Ibid. p. 21. §XVI. *Hos igitur ad orbitam cometæ mense martio hujus anni observati determinandam accommodabo. Observationes autem, quibus ad hoc institutum utar, mecum communicatæ sunt a Celeberrimo Astronomo De Lisle, quas sunimacura in observatorio Petropolitano habuit.*

[127] Ibid. p. 11

[128] Ibid. *Tabula motus cometae in parabola.*

Consider a comet (M) moving on a parabolic orbit, as shown in Fig. 23. The letter S denotes the Sun and P the perihelion of the orbit. The image on the left is from Euler, which we redraw on the right to show additional orbit parameters of the comet.

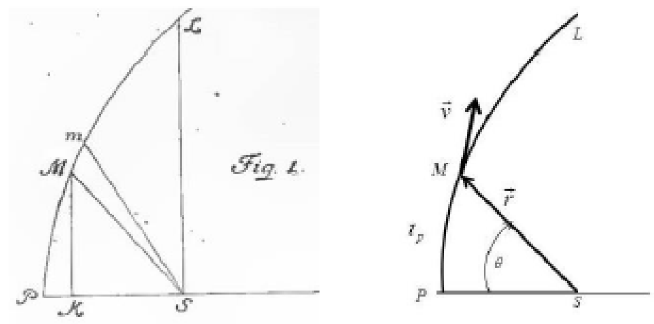

Fig. 23. Comet at M subject to the gravitational field of the Sun at S.

The Keplerian equation for a conic section with origin at a focus S relates the true anomaly θ and the position vector r is given by Eq. (3.8). As we shall show in Chapter 9, the parameter p of the orbit is related to the angular momentum h as

$$p = \frac{h^2}{GM} = \frac{h^2}{\mu}$$

where $\mu = GM$ is known as the standard gravitational parameter of the central body, the product of the universal gravitational constant G and the mass M of the body. See Table A.2.

Hence, the position of the orbiting body is given by

$$r = \frac{p}{1 + e\cos\theta} = \frac{h^2}{\mu} \cdot \frac{1}{1 + e\cos\theta} \qquad (6.1)$$

where e denotes the eccentricity of the sonic section.

The true anomaly θ is an angular parameter that defines the position of a body moving along a Keplerian orbit. It is the angle between the direction of periapsis and the current position of the

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Spacecraft Orbit Mechanics

Ipse Pater statuit, quae vis coeli astra moveret. Euler

Two hundred years after Euler began to build the analytical foundation of astromechanics, humanity was ready to extend the orbit formulations of celestial bodies to human spaceflight. It was no longer sufficient to know how the bodies in the sky move; humans were yearning to explore space directly and visit other worlds. The American Apollo Program was conceived with the goal of landing the first humans on the Moon and returning them safely to Earth.[235]

When in 1969 Apollo 11 took the three astronauts on the first mission to land on the Moon, the Saturn 5 rocket first carried the crewed spacecraft to an altitude of 185 km (low Earth orbit, LEO), moving at a speed of 24,800 km/h (6.88 km/s) —just below orbital velocity. Then the third rocket stage fired briefly, enough to accelerate the spacecraft to the required speed. The rocket engine was shut down, allowing the vehicle to coast in that Earth orbit. After performing the required maneuvers to line up the spacecraft on the correct flight path, the third stage of the rocket was restarted, increasing the speed to about 10.86 km/s (39,100 km/h), the escape velocity at that altitude required to completely overcome the influence of Earth's gravity.

For three days, Apollo 11 coasted around the Earth, spiraling up on its way to the Moon, until the pull of our planet's gravity became weaker and the spacecraft slowed down. By the time Apollo 11 was about 346,000 km from Earth its speed had decreased considerably. Then the gravity of the Moon began to

[235] Apollo 11 was America's program that landed the first two humans on the Moon. The spacecraft was launched by a Saturn V rocket from Kennedy Space Center in Florida, on July 16 at 9:32 am EDT (13:32 UTC) and was the fifth manned mission of NASA's Apollo program.

pull the spacecraft in, increasing its velocity. Once in lunar orbit, the astronauts separated the lunar module, fired its descent stage, and began the landing maneuver. When the lander was about 1.5 meters from the lunar surface, the engine shut off, and the vehicle carrying two people touched the surface of the Moon. Figure 37 illustrates the overall trajectory of the Apollo spacecraft on the two-way trip to the Moon.

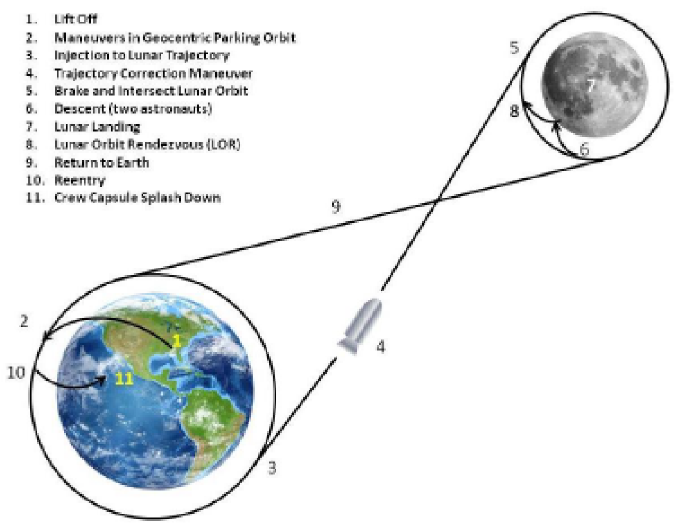
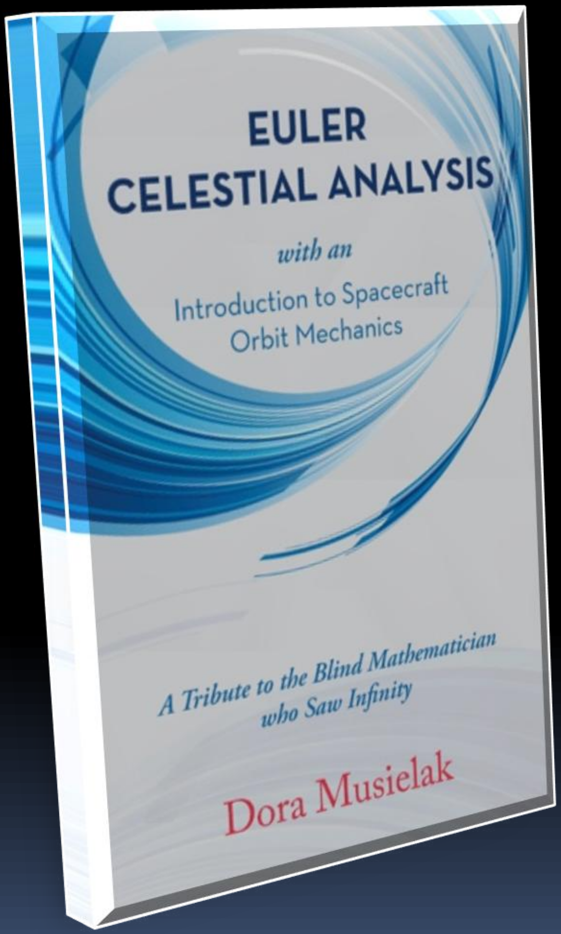


Fig. 37. Trajectory of the Apollo 11 Mission to the Moon.

It may seem easy, but the spaceflight plan that resulted in such a momentous milestone for humanity relied in mathematically precise calculations, simulations, and extensive planning, deriving the analysis from orbital mechanics and rocket science. Since the movement of every object in space is influenced by gravity and follows orbital paths—around planets, moons, and stars, spacecraft travel from a moving origin to moving targets in the Solar System.

Contact



Review, complimentary copies are available for instructors and students.

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